Chapter 3—Constructions: Part 3

In this lesson you will:

- construct the incenter, circumcenter, and orthocenter of a triangle
- make conjectures about the properties of the incenter and circumcenter of a triangle
- construct the centroid of a triangle
- make conjectures about the properties of the centroid of a triangle

*Add "concurrent lines," "median," "midsegment (of a triangle)," and "altitude (of a triangle)" to your dictionary.

Investigation 3.7: "Concurrence"

In this investigation you will discover that some special lines in a triangle have points of concurrency.

A.) Using Geometer's Sketchpad, make a triangle and construct the three angle bisectors for each triangle. Are they concurrent? _________ Change the shape of the triangle. Are the angle bisectors still concurrent? _________ Based on your observations, complete the conjecture below.

Angle Bisector Concurrency Conjecture (C-9)

The three angle bisectors of a triangle ____.

(Continue to add new conjectures to your conjecture list.)

B.) Using Geometer's Sketchpad, make a triangle and construct the perpendicular bisector for each side of the triangle. Complete the conjecture below based on your observations.

Perpendicular Bisector Concurrency Conjecture (C-10)

The three perpendicular bisectors of a triangle ____.

C.) Using Geometer's Sketchpad, make a triangle and construct the altitudes for each side of the triangle. Complete the conjecture below based on your observations.

Altitude Concurrency Conjecture (C-11)

The 3 altitudes (or the lines containing the altitudes) of a triangle ___________________________.

*Add “incenter,” “circumcenter,” and “orthocenter” to your dictionary.

D.) On your triangle in part B, compare the distances from the circumcenter to each of the 3 vertices. Are they the same? ______ Compare the distances from the circumcenter to each of the 3 sides. Are they the same? ______ Use your observations to finish the next conjecture.

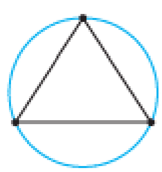
Circumcenter Conjecture (C-12)

The circumcenter of a triangle is ____.

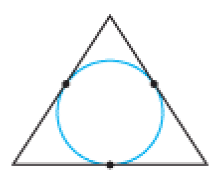
E.) On your triangles in part A, compare the distances from the incenter to each of the 3 sides. Are they the same? ______ Use your observations to state your next conjecture.

Incenter Conjecture (C-13)

The incenter of a triangle is ____.



Circumscribed circle
(inscribed triangle)



Inscribed circle
(circumscribed triangle)

*The point of concurrency of the perpendicular bisectors is the ___________ of a circle that circumscribes the triangle and thus is called the circumcenter of the triangle.

*The point of concurrency of the angle bisectors is the ___________ of a circle that is inscribesd in the triangle and thus is called the incenter of the triangle.

Investigation 3.8: "Are Medians Concurrent?"

A.) Using Geometer's Sketchpad, make a triangle and locate the midpoints of the three sides of each triangle below. Construct the medians (line segments connecting a vertex of a triangle to the midpoint of the opposite side). Complete the conjecture below based on your observations.

Median Concurrency Conjecture (C-14)

The three medians of a triangle ____.

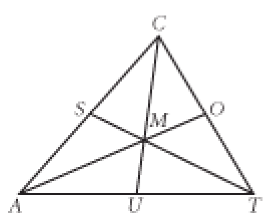
*The point of concurrency of the three medians is called the **centroid**. Add "centroid" to your dictionary.

Unfortunately, we don't have time to do more investigations to discover more interesting facts about triangles, but there are two more conjectures you need to know. Please also add them to your conjecture list.

Centroid Conjecture (C-15)

The centroid of a triangle divides each median into two parts so that the distance from the centroid to the vertex is twice the distance from the centroid to the midpoint of the opposite side.

•Example 1:



MU = 2 CM = ______

AM = 6 MO = ______

ST = 12 SM = ______ TM = _______

Center of Gravity Conjecture (C-16)

The centroid of a triangle is the center of gravity (balancing point) of the triangular region.

⇒**ASSIGNMENT**: ____