Lesson 11.1: Similar Polygons

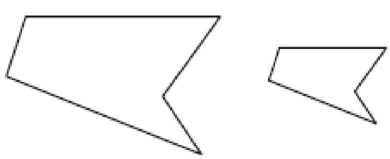
In this lesson you will:

- learn what it means for two figures to be similar
- use the definition of similarity of find missing measures in similar polygons
- explore dilations of figures on a coordinate plane

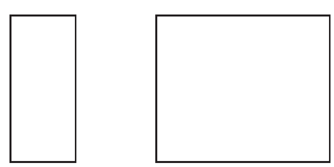
You know that figures that have the same shape and size are ______________ figures. Figures that have the same shape but not necessarily the same size are ____________ figures. To say that two figures have the same shape but not the same size is not, however, a precise definition of similarity.

Is your reflection in a fun-house mirror similar to a regular photograph of you? The images have a lot of features in common, but they are not mathematically similar. In mathematics, you can think of similar shapes as enlargements or reductions of each other with no irregular ______________.

Are all rectangles similar? _____ They have common characteristics, but they are not all similar. That is, you could not enlarge or reduce a given rectangle to fit perfectly over every other rectangle.



These pentagons are similar.

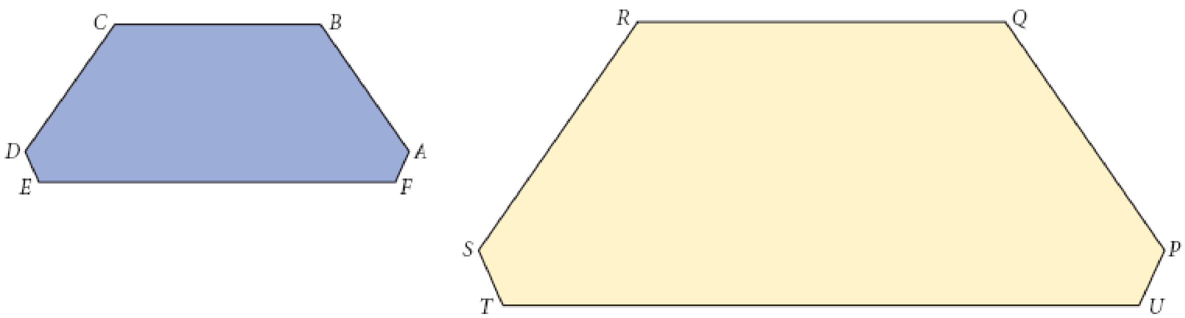


These rectangles are *not* similar.

Let's explore what makes polygons similar.

Investigation 11.1.1: “What Makes Polygons Similar?”

Hexagon *PQRSTU* is an enlargement of hexagon *ABCDEF*—they are similar.



A.) Copy hexagon *ABCDEF* onto patty paper. Compare its corresponding angles to hexagon *PQRSTU*. How do the corresponding angles compare?

B.) Measure all sides in both hexagons to the nearest 0.1 cm. Label the measurements next to the sides.

C.) Find the ratios of the lengths of corresponding sides.

$\frac{AB}{PQ} \approx$ $\frac{BC}{QR} \approx$ $\frac{CD}{RS} \approx$

$\frac{DE}{ST} \approx$ $\frac{EF}{TU} \approx$ $\frac{FA}{UP} \approx$

D.) How do the ratios of the corresponding sides (in part C) compare?

E.) Calculate and compare these side length ratios within each polygon: $\frac{AB}{BC}$ with $\frac{PQ}{QR}$ and $\frac{EF}{CD}$ with $\frac{TU}{RS}$.

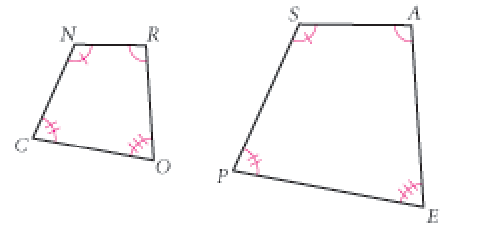
$\frac{AB}{BC} \approx$	$\frac{PQ}{QR} \approx$

$\frac{EF}{CD} \approx$	$\frac{TU}{RS} \approx$

What do you notice?

From the investigation, you should be able to state a mathematical definition of similar polygons. Two polygons are similar if and only if the corresponding angles are ______________ and the corresponding sides are ________________________. Similarity is the state of being similar.

*Add “similar” to your dictionary.



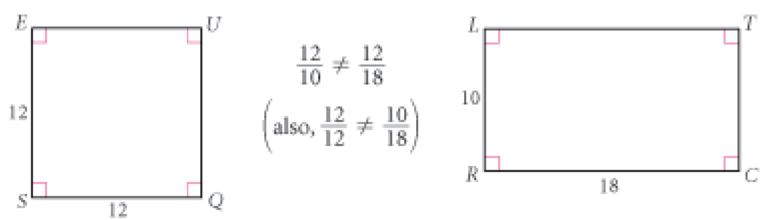
CORN is similar to PEAS if and only if $\frac{CO}{PE} = \frac{OR}{EA} = \frac{RN}{AS} = \frac{NC}{SP}$.

The statement *CORN* ~ *PEAS* says that quadrilateral *CORN* is similar to quadrilateral *PEAS*. Just as in statements of congruence, the order of the letters tells you which segments and which angles in the two polygons correspond.

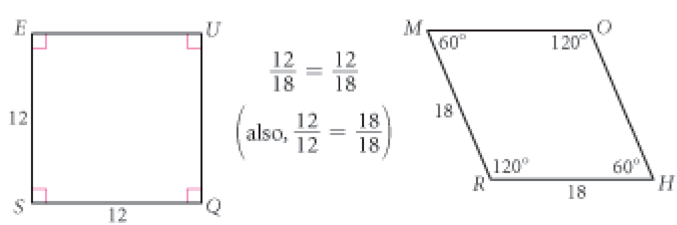
Notice also that the ratio of the lengths of any two segments in one polygon is equal to the ratio of the corresponding two segments in the similar polygon. For example, $\frac{CO}{OR} = \frac{PE}{EA}$ or $\frac{NR}{CO} = \frac{SA}{PE}$.

Do you need both conditions—congruent angles and proportional sides—to guarantee that the two polygons are similar? Let’s look at the following two examples.

In the figures below, corresponding angles of square *SQUE* and rectangle *RCTL* are congruent, but their corresponding sides are not ____________________.

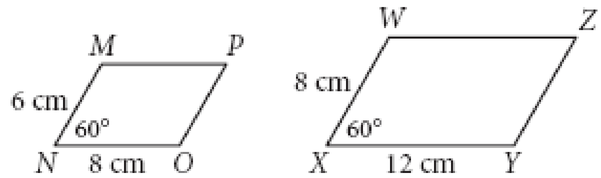


In the next set of figures, corresponding sides of square *SQUE* and rhombus *RHOM* are proportional, but their corresponding angles are not _______________.

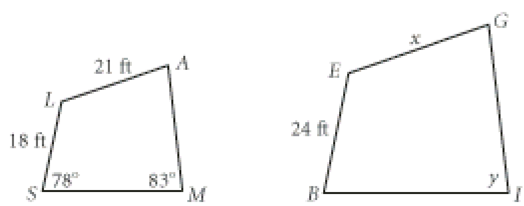


Clearly, neither pair of polygons is similar. Therefore, BOTH conditions—congruent angles and proportional sides—must be met to guarantee that two polygons are similar.

•Example 1: Determine whether parallelogram *MNOP* is similar to parallelogram *WXYZ*.



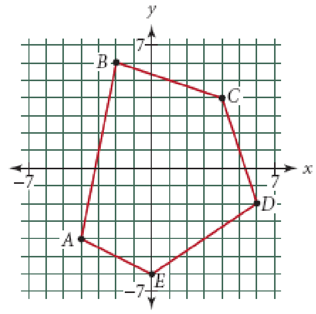
•Example 2: *SMAL* ~ *BIGE*
Find *x* and *y*.



Earlier in this book you worked with translations, rotations, and reflections. These rigid transformations preserve both size and __________—the images are congruent to the original figures. One type of nonrigid transformation is called a dilation.

*Add “dilation” and “scale factor” to your dictionary.

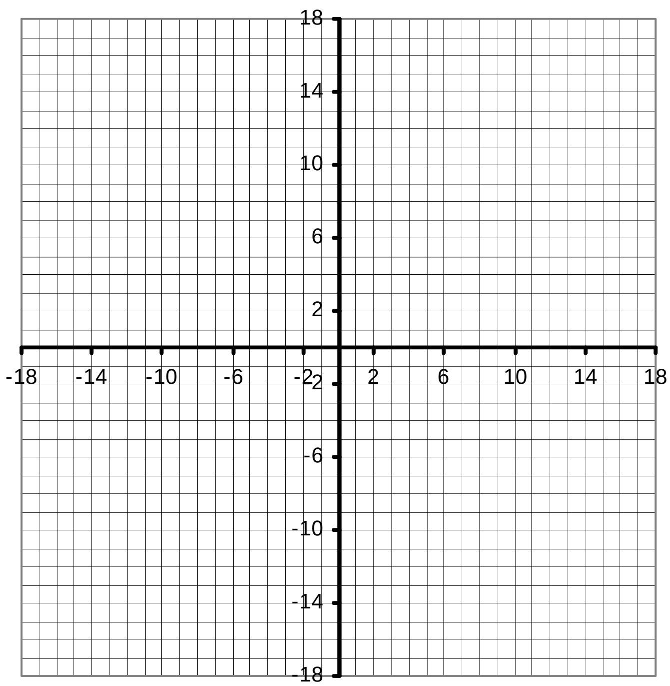
Investigation 11.1.2: “Dilations on the Coordinate Plane”



In this investigation you will examine the effects of dilating a pentagon about the origin. Have each member of your group choose a different scale factor from these choices: ½, ¾, 2, or 3.

A.) Copy this pentagon onto the grid below.
B.) List the coordinates of the vertices.

A(____,____); B(____,____); C(____,____); D(____,____); E(____,_____)



C.) Multiply the coordinates of the vertices by your scale factor, which is ______. List the new coordinates.

A’(____,____)

B’(____,____)

C’(____,____)

D’(____,____)

E’(____,____)

D.) Plot these new coordinates on the grid and connect them in a different color than the original. This new pentagon is a dilation of the original pentagon.

E.) How do the corresponding angles of the two pentagons compare?

F.) Compare the corresponding sides. The length of each side of the new pentagon is how many times as long as the length of the corresponding side of the original pentagon?

G.) Are the two pentagons similar?

H.) Based on your observations, complete the following conjecture:

Dilation Similarity Conjecture (C-90)

If one polygon is a dilated image of another polygon, then the polygons are _______________.

⇒**ASSIGNMENT**: ____