Lesson 11.2: Similar Triangles

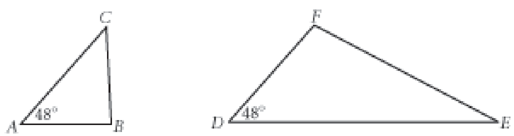
In this lesson you will:

- learn shortcuts for determining whether two triangles are similar

In Lesson 11.1, you concluded that you must know about both the angles and the sides of two quadrilaterals in order to make a valid conclusion about their similarity.

However, triangles are unique. Recall that from Chapter 4 that you found four shortcuts for triangle congruence: SSS, SAS, ASA, and SAA. Are there shortcuts for triangle similarity as well? Let's first look for shortcuts using only angles.

The figures below illustrate that you can cannot conclude that two triangles are similar given that only one set of corresponding angles are congruent.



$\angle A \cong \angle D$, but ΔABC is not similar to ΔDEF or ΔDFE.

How about two sets of congruent angles?

Investigation 11.2.1: "Is AA a Similarity Shortcut?"

If two angles of one triangle are congruent to two angles of another triangle, must the two triangles be similar? Open the Dynamic Geometry Exploration on Similar Triangles. Look at the first sketch.

A.) If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, what is true about $\angle C$ and $\angle F$? Why?

B.) Compare the ratios of the corresponding sides. Is $AB/DE = BC/EF = CA/FD$? ________

C.) Based on your results in parts A and B, are the triangles similar? ______

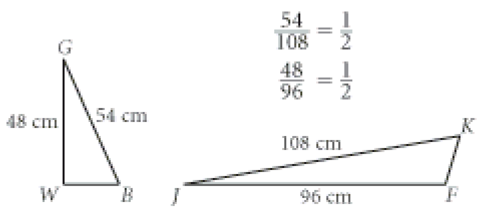
D.) Based on your observations, complete the conjecture below.

AA Similarity Conjecture (C-91)

If _______ angles of one triangle are congruent to _______ angles of another triangle, then the triangles are ________________.

E.) Why is there no need to investigate AAA, ASA, or SAA as a similarity shortcut?

Now let's look for shortcuts for similarity that use only sides. The figures below illustrate that you cannot conclude that two triangles are similar given that two sets of corresponding sides are proportional.



$\frac{GB}{JK} = \frac{GW}{JF}$, but ΔGWB is not similar to ΔJFK.

Investigation 11.2.2: "Is SSS a Similarity Shortcut?"

If three sides of one triangle are proportional to the three sides of another triangle, must the two triangles be similar? Look at the second sketch in the Dynamic Geometry Exploration on Similar Triangles.

A.) Compare the measures of the corresponding angles of the two triangles when all three sides are proportional. What do you notice?

B.) Based on your results in parts A and B, are the triangles similar? ______

C.) Based on your observations, complete the conjecture below.

SSS Similarity Conjecture (C-92)

If the three sides of one triangle are proportional to the three sides of another triangle, then the triangles are ________________.

So SSS and AA (which encompasses AAA, ASA, and SAA) are shortcuts for triangle similarity. That leaves SAS and SSA as possible shortcuts to consider.

Investigation 11.2.3: "Is SAS a Similarity Shortcut?"

Use the third sketch of the Dynamic Geometry Exploration to determine whether two triangles are similar if they have two pairs of sides proportional and the pair of included angles equal in measure.

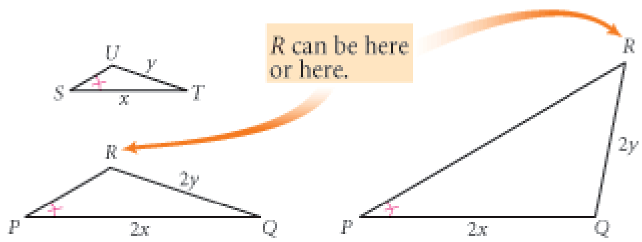
A.) In the triangles, two sets of corresponding sides are proportional (*AB* and *DE*, and *AC* and *DF*). When you make included $\angle D$ congruent to included $\angle B$, are the two triangles similar? _______

B.) Based on your observations, complete the conjecture below.

SAS Similarity Conjecture (C-93)

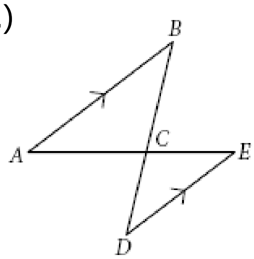
If two sides of one triangle are proportional to two sides of another triangle and the included angles are ________________, then the triangles are _______________.

One question remains: Is SAA a shortcut for similarity? Recall from Chapter 4 that SSA did not work for congruence because you could create two different triangles. Those two different triangles were neither congruent nor similar. So, no, SSA is not a shortcut for similarity.

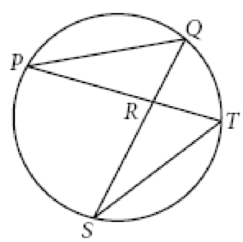


•Example 1: Identify similar triangles, and explain why they are similar.

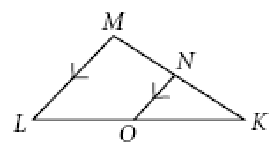
a.)



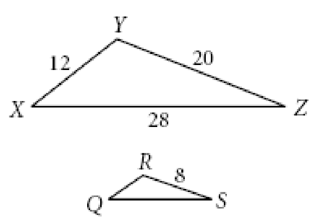
b.)



c.)



•Example 2:



$\triangle XYZ \sim \triangle QRS$

$\angle Q \cong$ _____

$QR =$ _____

$QS =$ _____

⇒**ASSIGNMENT**: ____