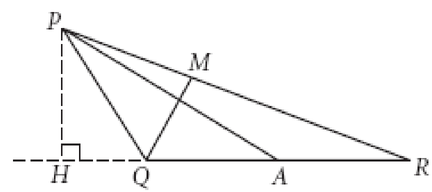
Lesson 11.4: Corresponding Parts of Similar Triangles

In this lesson you will:

- investigate the relationship between corresponding altitudes, corresponding medians, and corresponding angle bisectors of similar triangles
- discover a proportional relationship involving angle bisectors

Is there more to similar triangles than just proportional side lengths and congruent angles? For example, there are relationships between the lengths of corresponding altitudes, corresponding medians, or corresponding angle bisectors in similar triangles.

First, let's review what altitudes, medians, and angle bisectors are.



♦ What is the altitude of ΔPQR? _______

♦ If $\overline{PA}$ is a median, what can we conclude based on the definition of a median?

♦ If $\overline{QM}$ is an angle bisector, what can we conclude based on the definition of an angle bisector?

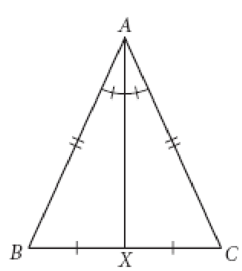
We know that in similar triangles that the side lengths are proportional. Intuitively, do you think that the lengths of corresponding altitudes, medians, and angle bisectors are also proportional? ________

Proportional Parts Conjecture (C-94)

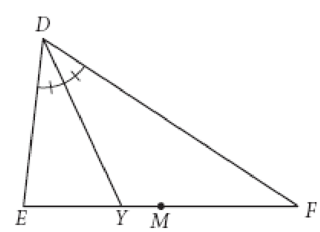
If two triangles are similar, then the lengths of the corresponding _________________, _________________, and ______________________________ are ____________________ to the lengths of the corresponding sides.

Now we'll look at another proportional relationship involving an angle bisector of a triangle.

If a triangle is isosceles, the bisector of the vertex angle divides the opposite sides into equal parts. That is, the angle bisector is also a median; and, in fact, it is also the altitude. (Vertex Angle Bisector Conjecture C-27) However, as the triangle on the right shows, this is not true for all triangles.



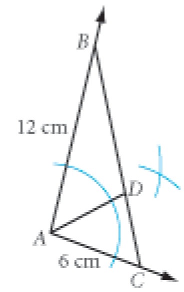
$\overline{AX}$ is an angle bisector.
Point X is the midpoint of $\overline{BC}$.



$\overline{DY}$ is an angle bisector.
Point M is the midpoint of $\overline{EF}$.

The angle bisector does, however, divide the opposite side in a particular way.

Investigation 11.4.2: "Opposite Side Ratios"



A.) Draw any angle below. Label it *A*. (Make sure *A* is positioned near the bottom of the available space and extend the rays at least 12 cm.)

B.) On one ray, locate point *C* so that *AC* is 6 cm. Locate point *B* on the other ray so that *AB* is 12 cm. Draw $\overline{BC}$ to form ΔABC.

C.) Construct the angle bisector of $\angle A$. Locate point *D* where the bisector intersects side $\overline{BC}$.

D.) Measure *CD* and *BD*. How do they compare?

E.) Calculate and compare the ratios $\frac{CA}{BA}$ and $\frac{CD}{BD}$.

F.) Draw another angle below. Label it *X*.

G.) On one ray, locate point *Z* so that *XZ* is 4 cm. Locate point *Y* on the other ray so that *XY* is 6 cm. Draw $\overline{YZ}$ to form ΔXYZ.

H.) Construct the angle bisector of $\angle X$. Locate point *D* where the bisector intersects side $\overline{YZ}$.

I.) Measure *ZD* and *YD*. How do they compare?

J.) Calculate and compare the ratios $\frac{ZX}{YX}$ and $\frac{ZD}{YD}$.

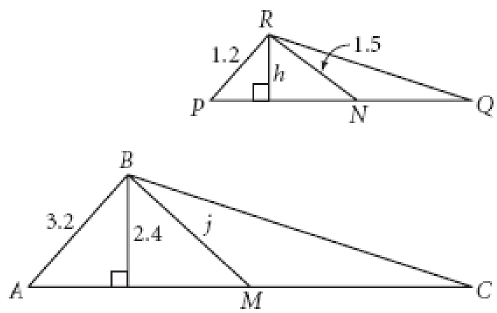
K.) Based on your results, complete the following conjecture:

Angle Bisector/Opposite Side Conjecture (C-95)

A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the two ____________ forming the angle.

•Example 1: $\Delta ABC \sim \Delta PRQ$. *M* and *N* are midpoints. Find *h* and *j*.

h = ______ cm j = ______ cm



•Example 2: $\Delta ABC \sim \Delta WXY$.

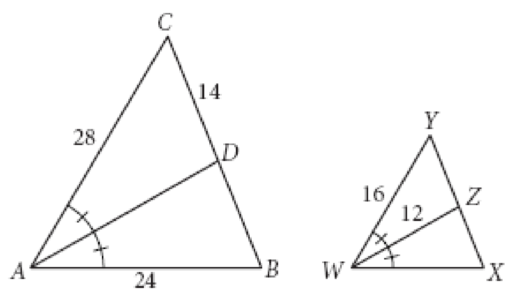
WX = ______ cm

DB = ______ cm

XZ = ______ cm

AD = ______ cm

YZ = ______ cm



⇒**ASSIGNMENT**: ____