Lesson 11.5: Proportions with Area

In this lesson you will:

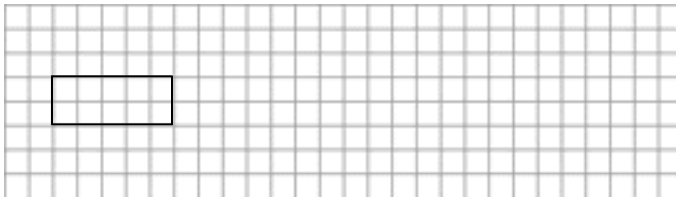
- discover the relationship between the areas of similar figures

Suppose you painted the ceiling in small room of your house. You used one small can of paint. Now you move onto the ceiling of a larger room. This larger ceiling is twice the corresponding dimensions of the smaller ceiling. You buy two small cans of paint but soon realize that the two cans only cover a small portion of this larger ceiling. In this lesson you will discover why.

Investigation 11.5: "Area Ratios"

In this investigation you will find the relationship between areas of similar figures.

A.) The rectangle below has a base of 5 units and a height of 2 units.
What is its area? _____ sq. units



B.) If both sides lengths increased by a scale factor of two, what would be the new base and height of the rectangle? _____ units by _____ units

C.) Draw this new rectangle on the grid to the right of the original rectangle.

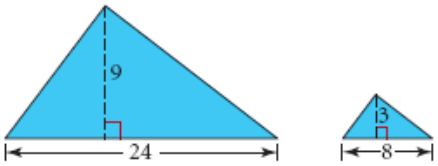
D.) What is the area of the new rectangle? _____ sq. units

E.) How many copies of the original rectangle would you need to fill the larger rectangle? _____

F.) Find the area of these two similar triangles.

Area of smaller triangle = _____ units^2

Area of larger triangle = _____ units^2

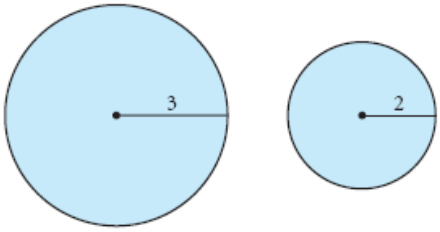


G.) Write the ratio of the smaller area to the larger area as a fraction and reduce.

H.) Write a ratio of side lengths of the smaller triangle to the larger triangle and reduce.

I.) How do the two ratios compare?

J.) Find the exact area of these two circles in terms of π, not as approximate decimals.



Area of smaller circle = _____ units^2

Area of larger circle = _____ units^2

K.) Write the ratio of the smaller area to the larger area as a fraction and reduce.

Write the ratio of the smaller radius to the larger radius.

L.) How do the two ratios compare?

M.) Compare your results with your group and look for a pattern. This relationship between the ratio of corresponding sides (or radii) and the ratio of areas can be generalized to all similar polygons because all polygons. You should be ready to complete the conjecture below.

Proportional Areas Conjecture (C-96)

If corresponding side lengths of two similar polygons or radii of two circles compare in the ratio $\frac{m}{n}$, then their areas compare in the ratio _____.

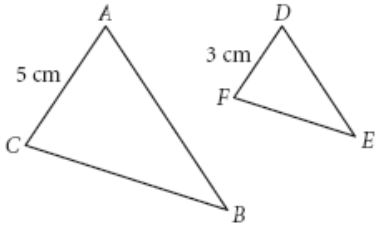
The reason behind the Proportional Areas Conjecture is that area is a two-dimensional measure. Calculating area involves multiplying two linear measures, such as base and height. So, if a rectangle has base *b* and height *h*, then its area is *bh*. If the base and height are each multiplied by 2, then the area of the new rectangle is $2b \cdot 2h = 4bh$. This is ____ times the area of the original rectangle. Similarly, if the base and height are each multiplied by 3, then the area of the new rectangle is $3b \cdot 3h =$ ___*bh*. This is ____ times the area of the original rectangle.

In Chapter 8, we also looked at surface areas. Do you think that the Proportional Area Conjecture would also apply to surface areas? _____

So, thinking back to the ceiling painting problem at the beginning of this lesson…how many small cans of paint should you have bought? _____

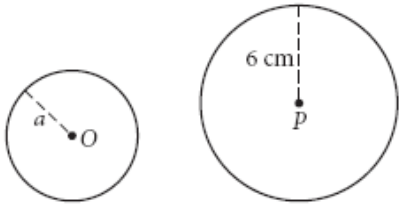
•Example 1: $\triangle ABC \sim \triangle DEF$. Area of $\triangle ABC = 15$ cm^2.

Area of $\triangle DEF =$ _____



•Example 2: $\frac{\text{Area of circle } O}{\text{Area of circle } P} = \frac{4}{9}$.

$a =$ _____



•Example 3: If you need 3 oz of shredded cheese to cover a medium 12 in. diameter pizza, how much shredded cheese would you need to cover a large 16 in. diameter pizza?

⇒**ASSIGNMENT**: _____