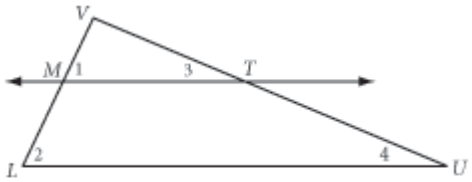
Lesson 11.7: Proportional Segments Between Parallel Lines

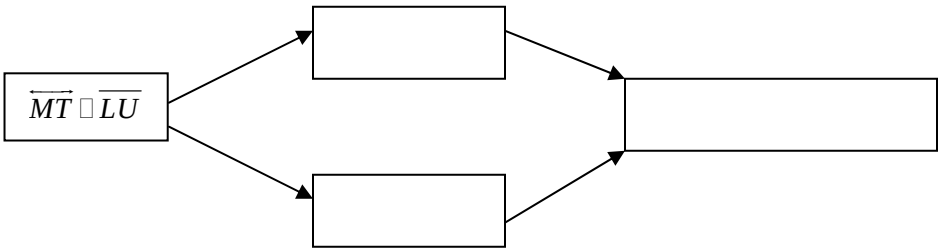
In this lesson you will:

- explore the relationships in the lengths of segments formed when one or more lines parallel to one side of a triangle intersect the other two sides

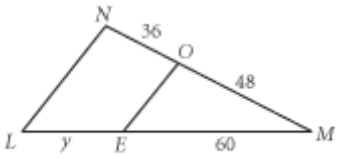
In the figure below, $\overleftrightarrow{MT} \parallel \overline{LU}$. Is ΔLUV similar to ΔMTV? _____ A short proof can support this observation.

Given: ΔLUV with $\overleftrightarrow{MT} \parallel \overline{LU}$
Show: $\Delta LUV \sim \Delta MTV$

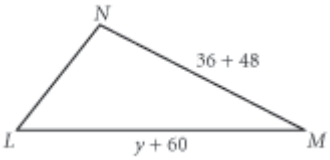


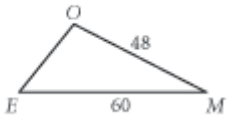


•Example 1: $\overline{EO} \parallel \overline{LN}$ Find y.



Hint: Separate ΔEMO and ΔLMN so that you can see the proportional relationships more clearly. Is $\Delta EMO \sim \Delta LMN$? _____





Investigation 11.7.1: "Parallels and Proportionality"

In this investigation we'll look at the ratios of segments that have been cut by parallel lines.

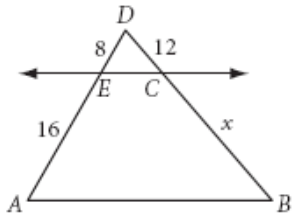
A.) Separate each figure below into two triangles. Then find x and numerical values for the given ratios.

i.) $\overleftrightarrow{EC} \parallel \overline{AB}$

$x =$ ____

$\frac{DE}{AE} =$

$\frac{DC}{BC} =$

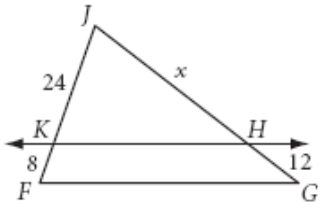


ii.) $\overleftrightarrow{KH} \parallel \overline{FG}$

$x =$ ____

$\frac{JK}{KF} =$

$\frac{JH}{HG} =$

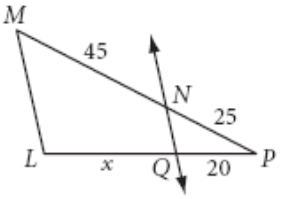


iii.) $\overleftrightarrow{QN} \parallel \overline{LM}$

$x =$ ____

$\frac{PQ}{QL} =$

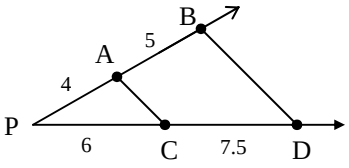
$\frac{PN}{MN} =$



B.) What do you notice about the ratios of the lengths of the segments that have been cut by the parallel lines?

Is the converse true? That is, if a line divides two sides of a triangle proportionally, is it parallel to the third side? Let's see.

C.) Draw an acute angle, *P*. (Make sure point *P* is positioned near the bottom right of the available space below and extend the rays at least 14 cm.)



D.) Beginning at point *P*, use your ruler to mark off lengths of 4 cm and 5 cm on one ray. Label the points *A* and *B*.

E.) On the other ray, mark off lengths of 6 cm and 7.5 cm. Label the points C and D. Notice that $\frac{4}{5} = \frac{6}{7.5}$.

F.) Draw $\overline{AC}$ and $\overline{BD}$.

G.) $\angle PAC$ and $\angle PBD$ are _____ angles.

H.) With a protractor, measure $\angle PAC$ and $\angle PBD$. What is true about the measures?

Are $\overline{AC}$ and $\overline{BD}$ parallel? _____

I.) Based on your observations, complete the conjecture:

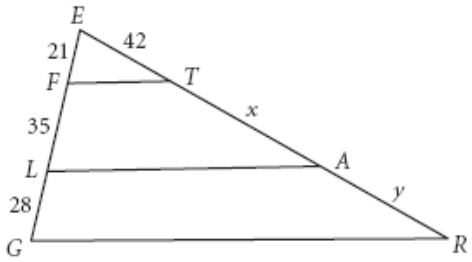
Parallel/Proportionality Conjecture (C-98)

If a line parallel to one side of a triangle passes through the other two sides, then it divides the other two sides _____. Conversely, if a line cuts two sides of a triangle proportionally, then the line is _____ to the third side.

Investigation 11.7.2: "Extended Parallel/Proportionality"

A.) Use the Parallel/Proportionally Conjecture to find each missing length.

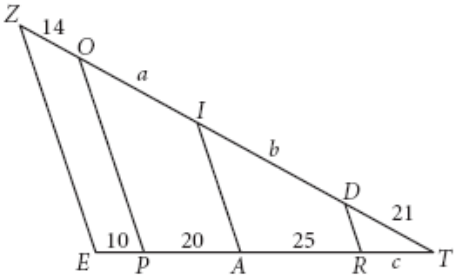
i.) $\overline{FT} \parallel \overline{LA} \parallel \overline{GR}$



$x =$ _____ $y =$ _____

Is $\frac{FL}{LG} = \frac{TA}{AR}$? _____

ii.) $\overline{ZE} \parallel \overline{OP} \parallel \overline{IA} \parallel \overline{DR}$



$a =$ _____ $b =$ _____ $c =$ _____

Is $\frac{DI}{IO} = \frac{RA}{AP}$? _____ Is $\frac{IO}{OZ} = \frac{AP}{PE}$? _____

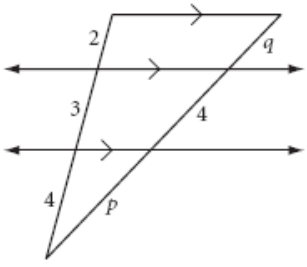
B.) Compare your results with your group. Then complete the conjecture below.

Extended Parallel/Proportionality Conjecture (C-99)

If two or more lines pass through two sides of triangle parallel to the third side, then they divide the two sides _____.

•Example 2: $p =$ _____

$q =$ _____



⇒**ASSIGNMENT**: _____