Lesson 12.1: Trigonometric Ratios

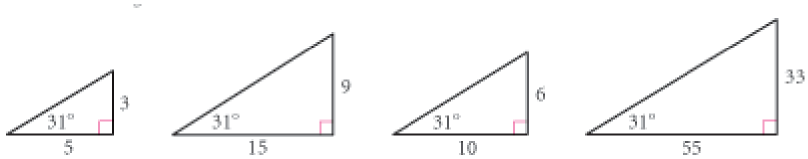
In this lesson you will:

- learn about the trigonometric ratios sine, cosine, and tangent
- use trigonometric ratios to find unknown side lengths in right triangles
- use inverse trigonometric functions to find unknown angle measures in right triangles

*Add “trigonometry,” “opposite leg,” “adjacent leg,” “tangent,” “sine,” “cosine,” and “inverse cosine, sine, or tangent” to your dictionary.

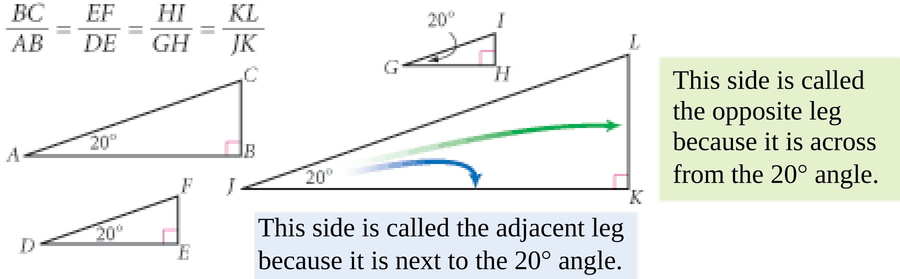
Trigonometry is the study of the relationships between the sides and angles of _____________. In this lesson you will discover some of these relationships for right triangles.

When studying right triangles, early mathematicians discovered that whenever the ratio of the shorter leg’s length to the longer leg’s length was close to a specific fraction, the angle opposite the shorter leg was close to a specific measure. They found this (and its converse) to be true for all similar right triangles. For example, in every right triangle in which the ratio of the shorter leg’s length to the longer leg’s length is $\frac{3}{5}$, the angle opposite the shorter leg is almost exactly ____°.



What early mathematicians discovered is supported by what you know about similar triangles. If two right triangles each have an acute angle of the same measure, then the triangles are similar by the ______ Similarity Conjecture. And if the triangles are similar, then corresponding sides are ___________________. For example, in the similar right triangles shown below, these proportions are true:

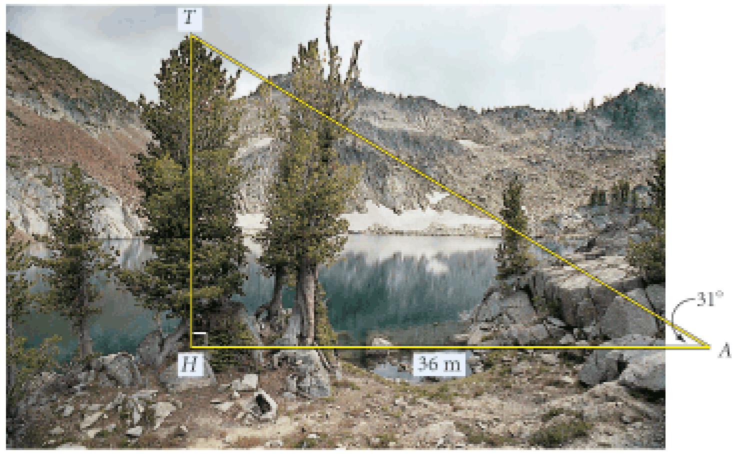
$$\frac{BC}{AB} = \frac{EF}{DE} = \frac{HI}{GH} = \frac{KL}{JK}$$

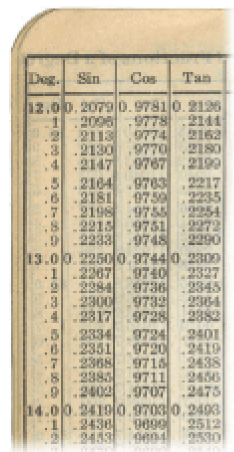


The ratio of the length of the opposite leg to the length of the adjacent leg in a right triangle came to be called the tangent of the angle.

In Chapter 11, you used mirrors and shadows to measure heights indirectly. Trigonometry gives you another indirect measuring method.

•Example 1: At a distance of 36 meters from a tree, the angle from the ground to the top of the tree is 31°. Find the height of the tree.



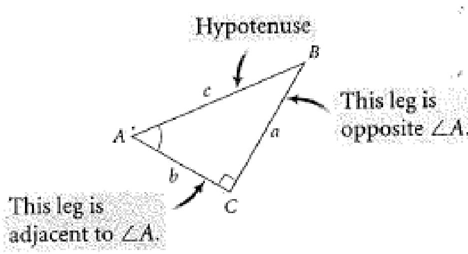


Deg.	Sin	Cos	Tan
12.0	0.2079	0.9781	0.2126
.1	.2096	.9778	.2144
.2	.2113	.9774	.2162
.3	.2130	.9770	.2180
.4	.2147	.9767	.2199
.5	.2164	.9763	.2217
.6	.2181	.9759	.2235
.7	.2198	.9755	.2254
.8	.2215	.9751	.2272
.9	.2233	.9748	.2290
13.0	0.2250	0.9744	0.2309
.1	.2267	.9740	.2327
.2	.2284	.9736	.2345
.3	.2300	.9732	.2364
.4	.2317	.9728	.2382
.5	.2334	.9724	.2401
.6	.2351	.9720	.2419
.7	.2368	.9715	.2438
.8	.2385	.9711	.2456
.9	.2402	.9707	.2475
14.0	0.2419	0.9703	0.2493
.1	.2436	.9699	.2512
.2	.2453	.9694	.2530

In order to solve problems like Example 1, early mathematicians made tables that related ratios of side lengths to angle measures. They named ___ possible ratios. You will work with three in this chapter:

- ♦Sine, abbreviated sin, is the ratio of the length of the _______________ leg to the length of the hypotenuse.
- ♦Cosine, abbreviated cos, is the ratio of the length of the _______________ leg to the length of the hypotenuse.
- ♦Tangent, abbreviated tan, is the ratio of the length of the _______________ leg to the length of the _____________ leg.

Trigonometric Ratios



For any acute angle A in a right triangle:	In $\triangle ABC$ above:
sine of $\angle A = \dfrac{\text{length of opposite leg}}{\text{length of hypotenuse}}$	$\sin A = \dfrac{a}{c}$
cosine of $\angle A = \dfrac{\text{length of adjacent leg}}{\text{length of hypotenuse}}$	$\cos A = \dfrac{b}{c}$
tangent of $\angle A = \dfrac{\text{length of opposite leg}}{\text{length of adjacent leg}}$	$\tan A = \dfrac{a}{b}$

Investigation 12.1: "Trigonometric Tables"

In this investigation you will make a table of trigonometric ratios for angles measuring 20° and 70°.

A.) Use your protractor to make a large right triangle *ABC* with $m\angle A = 20°$, $m\angle B = 90°$, and $m\angle C = 70°$.

B.) Measure *AB*, *AC*, and *BC* to the nearest 0.1 centimeter.

AB = _____ cm *AC* = _____ cm *BC* = _____ cm

C.) Use your side lengths and definitions of sine, cosine, and tangent to complete the table below. Round your ratios to 3 decimal places.

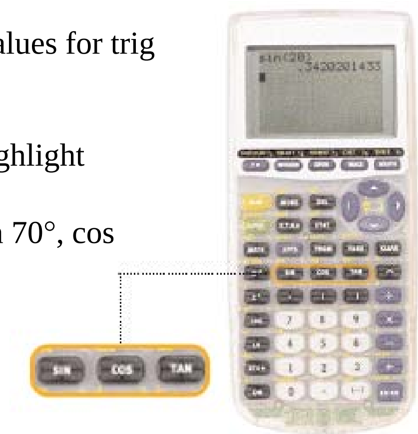
$m\angle A$	sin *A*	cos *A*	tan *A*
20°			

$m\angle C$	sin *C*	cos *C*	tan *C*
70°			

D.) Share your results with your group. (All group members should have ratios that are close to each other; if not, double check your ratios.)

E.) Discuss your results. What observations can you make about the trigonometric ratios you found? What is the relationship between the values of 20° and the values for 70°? Explain why you think these relationships exist.

Today, we no longer have to use trigonometric tables to find values for trig functions because our calculators have sin, cos, and tan keys.



F.) Change your calculator to "degree" mode. ($\boxed{\text{MODE}}$, highlight "Degree" instead of "Radian.")

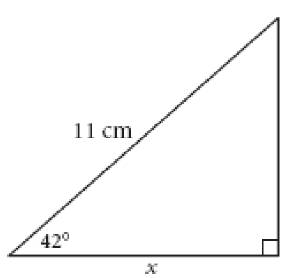
G.) Use your calculator to find sin 20°, cos 20°, tan 20°, sin 70°, cos 70°, and tan 70°.

sin 20° = _____, cos 20° = _____, tan 20° = _____,

sin 70° = _____, cos 70° = _____, tan 70° = _____.

H.) How do the trig ratios found by measuring the sides compare to the trig ratios that you found on the calculator?

•Example 2: Find the value of x.



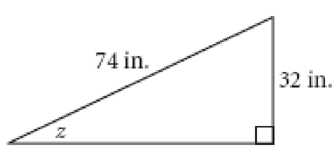
•Example 3: Find the length of the hypotenuse of a right triangle if an acute angle measures 20° and the leg opposite the angle measures 410 feet.

If you know the length of any two sides of a right triangle, you can use inverse trigonometric functions to find the angle measures. For instance, if you know the ratio of the legs in a right triangle, you can find the measure of one acute angle by using the inverse tangent, or $\tan^{-1}$, function. The inverse tangent of x is defined as the measure of the acute angle whose tangent is x. The tangent function and inverse tangent function undo each other. That is, $\tan^{-1}(\tan A) = A$ and $\tan(\tan^{-1} x) = x$.

•Example 4: A right triangle has legs of length 8 inches and 15 inches. Find the measure of the angle opposite the 8-inch leg.

You can also use inverse sine, or $\sin^{-1}$, and inverse cosine, or $\cos^{-1}$, to find angle measures.

•Example 5: Find the measure of angle z.



⇒**ASSIGNMENT**: ____