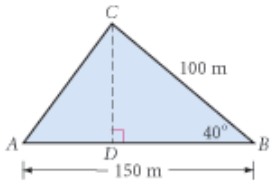
Lesson 12.3: The Law of Sines

In this lesson you will:

- find the area of a triangle when you know two side lengths and the measure of the included angle
- derive the Law of Sines, which relates the side lengths of a triangle to the sines of the angle measures
- use the Law of Sines to find an unknown side length of a triangle when you know the measures of two angles and one side or to find an unknown acute angle measure when you know the measures of two sides and one angle

So far you have used trigonometry only to solve problems with right triangles. But you can use trigonometry with any triangle. For example, if you know the measures of two angles and one side of a triangle, you can find the other two sides with a trigonometric property called the Law of Sines. The Law of Sines is related to the area of a triangle. Let's first see how trigonometry can help you find the area.

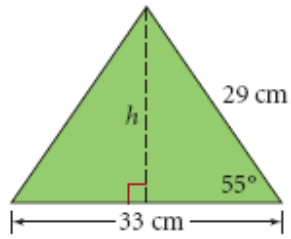
- Example 1: Find the area of ΔABC.



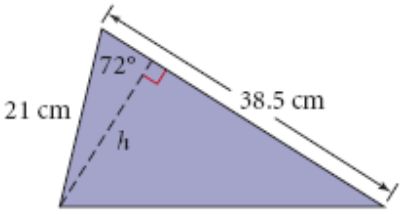
Investigation 12.3.1: "Area of a Triangle"

A.) Find the area of each triangle. (Use Example 1 as a guide.) Show your work!

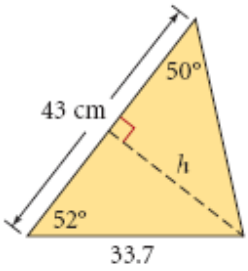
a.



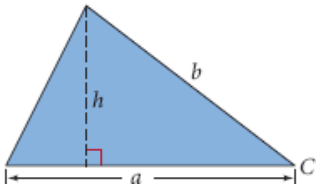
b.



c.



B.) Generalize your results to find the area of this triangle in terms of a, b, and $\angle C$, and complete the conjecture below.



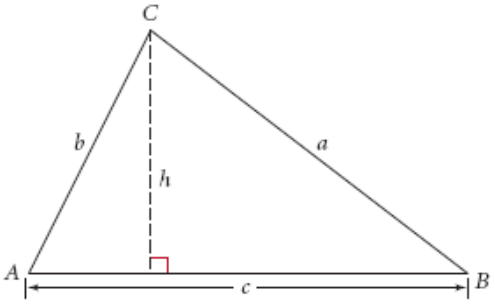
SAS Triangle Area Conjecture (C-100)

The area of a triangle is given by the formula $A =$ _____ where a and b are the lengths of two sides and C is the angle between them.

Now use what you have learned about finding the area of a triangle to derive the property called the Law of Sines.

Investigation 12.3.2: "The Law of Sines"

Consider ΔABC with height h.



A.) Find h in terms of a and the sine of an angle.

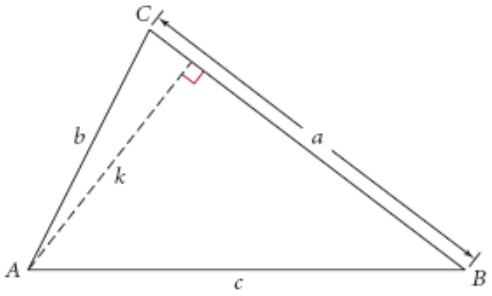
$h =$ _____

B.) Find h in terms of b and the sine of an angle.

$h =$ _____

C.) Use algebra to show that $\frac{\sin A}{a} = \frac{\sin B}{b}$.

Now consider the same ΔABC using a different height, k.



D.) Find k in terms of c and the sine of an angle.

$k =$ _____

E.) Find k in terms of b and the sine of an angle.

$k =$ _____

F.) Use algebra to show that $\frac{\sin B}{b} = \frac{\sin C}{c}$.

G.) Combine parts C and F to complete the conjecture below.

Law of Sines (C-101)

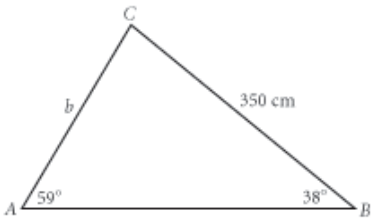
For a triangle with angles A, B, and C and sides of lengths a, b, and c (a opposite A, b opposite B, and c opposite C),

$$\frac{\sin A}{\quad} = \frac{\quad}{b} = \frac{\quad}{\quad}.$$

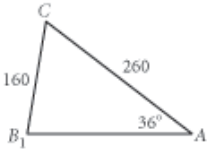
Did you notice that you used deductive reasoning rather than inductive reasoning to discover the Law of Sines?

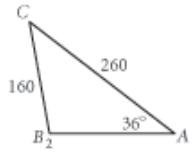
You can use the Law of Sines to find the lengths of a triangle's sides when you know one side's length and two angle measures.

•Example 2: Find the length of side $\overline{AC}$ in ΔABC.



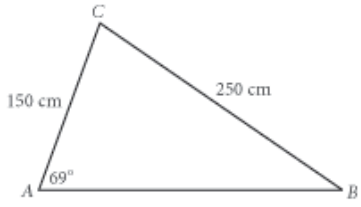
You can also use the Law of Sines to find the measure of a missing angle, but only if you know whether the angle is acute or obtuse. Recall from Chapter 4 that SSA failed as a congruence shortcut. For example, if you know in ΔABC that BC = 160 cm, AC = 260 cm, and $m\angle A$ = 36°, you would not be able to find $m\angle B$. There are two possible measures for $\angle B$, one acute and one obtuse.





Because you've defined trigonometric ratios only for acute angles, you'll be asked to find only acute angle measures.

•Example 3: Find the measure of acute angle *B* in ΔABC.



⇒**ASSIGNMENT**: _____