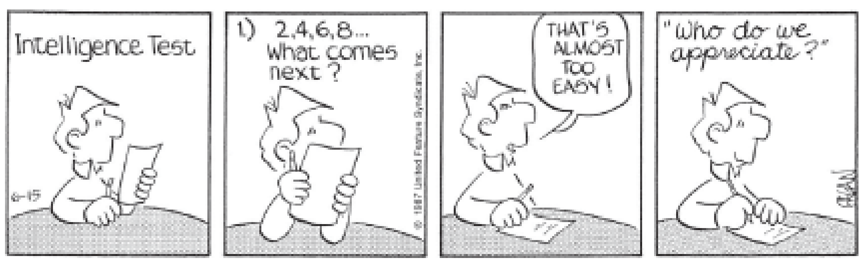
# Lesson 2.2: Finding the nth Term

In this lesson you will:

- learn how to write function rules for number sequences with a constant difference
- write a rule to describe a geometric pattern
- learn why a rule for a sequence with a constant difference is called a linear function

Consider the sequence 20, 27, 34, 41, 48, 55, 62, … Notice that the difference between any two consecutive terms is ____. We say that this sequence has a _____________ ________________ of 7. To find the next two terms in the sequence, you could add 7 to the last term to get 69, and then add 7 to 69 to get 76. But what if you wanted to find the 200th term? It would take a long time to list all the terms. If you could find a rule for calculating the *n*th term of the sequence for any number *n*, you could find the 200th term without having to list all the terms before it. This rule is called the _____________ rule. In the investigation you will learn (actually review!) a method for writing a rule for any sequence that has a constant difference.



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Investigation 2.2: "Finding the Rule"

A.) Complete each table below. Find the difference between consecutive values.

Differences:

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| *n* – 5 | -4 | -3 | -2 | | | | | |

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 4*n* – 3 | 1 | 5 | 9 | | | | | |

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| -2*n* + 5 | 3 | 1 | -1 | | | | | |

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 3*n* – 2 | 1 | 4 | 7 | | | | | |

| *n* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| -5*n* + 7 | 2 | -3 | -8 | | | | | |

B.) Did you spot the pattern? If a sequence has constant difference 4, then the number in front of the *n* (the coefficient of *n*) is _____. In general, if the difference between the values of consecutive terms of a sequence is always the same, say *m* (a constant), then the coefficient of *n* in the formula is ______.

Let's return to the sequence at the beginning of the lesson.

| Term | 1 | 2 | 3 | 4 | 5 | 6 | 7 | … | *n* |
|---|---|---|---|---|---|---|---|---|---|
| Value | 20 | 27 | 34 | 41 | 48 | 55 | 62 | … | |

+7

The constant difference is 7, so you know part of the rule is 7*n*. How do you find the rest of the rule?

C.) The first term ($n = 1$) of the sequence is 20, but if you apply the part of the rule you have so far, using $n = 1$, you get $7n = 7(1) = 7$, not 20. So how should you fix the rule? How can you get from 7 to 20? What is the rule for this sequence?

D.) Check your rule by trying the rule with other terms in the sequence.

•Example 1: Find the rule for the sequence 7, 2, -3, -8, -13, -18, …

The difference between the terms is always _____. So the rule is ___*n* + something. To find the unknown "something" (represented by *c*) replace the n in the rule with a term number. Try n = 1 and set the expression equal to 7 (because that is what the first term is equal to). Solve for c.

$$-5(1) + c = 7$$

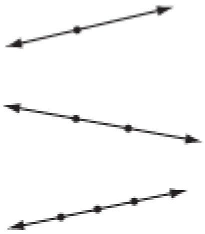
Rules for sequences can be expressed using **function notation**. For this example, $f(n)$ = ___________. In this case, function *f* takes an input value *n*, multiplies it by –5, and adds ____ to produce an output value.

You can find the value of any term in the sequence by substituting the term number for *n* into the function. To find the 20th term of this sequence, for instance, substitute 20 for *n*.

$f(20) = -5(20) + 12 =$ _____

Rules that generate a sequence with a constant difference are **linear functions**.

•Example 2: If you place 200 points on a line, into how many non-overlapping rays and segments does it divide the line?



| Points dividing the line | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| Non-overlapping rays | 2 | 2 | 2 | | | | … | | … | |
| Non-overlapping segments | 0 | 1 | 2 | | | | … | | … | |
| Total | 2 | 3 | 4 | | | | … | | … | |

⇒**ASSIGNMENT**: ______________________________________________