Lesson 2.3: Mathematical Modeling

In this lesson you will:

- attempt to solve a problem by acting it out
- create a mathematical model for a problem
- learn about triangular numbers and the formula for generating them

When you represent a situation with a graph, diagram, or equation, you are creating a **mathematical model**. Suppose you throw a ball straight up into the air with an initial velocity of 60 ft/s. You may recall from algebra that if you release the ball from a height of 5 ft, then the height *h* of the ball after *t* seconds can be modeled with the equation $h = -16t^2 + 60t + 5$. Once you have created a model, you can use it to make predictions. For example, you could use the equation or graph to predict the height of the ball after 2 seconds or to predict when the ball will hit the ground.

Investigation 2.3: "Party Handshakes"

Each of 30 people at a party shook hands with everyone else. How many handshakes were there altogether?

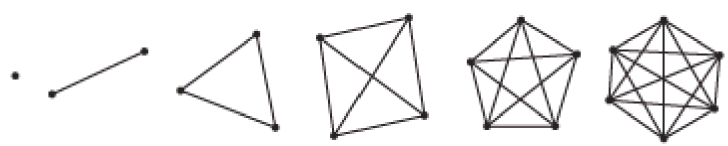
A.) Act out this problem with members of your group. Collect data for "parties" of one, two, three, and four people, and record your results in the table below.

People	1	2	3	4	…	30
Handshakes	0	1			…	?

B.) Look for a pattern. Can you generalize from your pattern to find the 30th term?

Acting out a problem is a powerful problem solving strategy that can give you important insight into a solution. Were you able to make a generalization from just four terms? If so, how confident are you of your generalization? To collect more data, you can ask more classmates to join your group. You can see, however, that acting out a problem sometimes has its practical limitations. That's when you can use mathematical models.

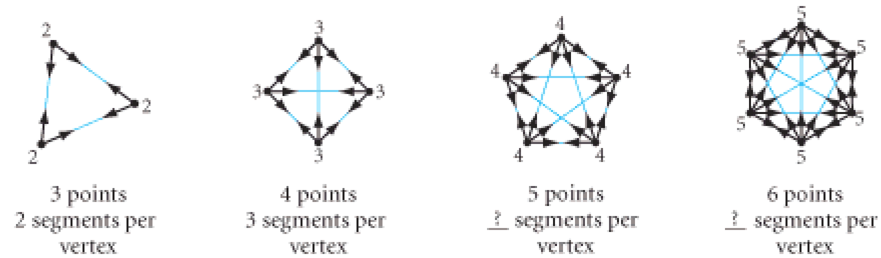
C.) Model this problem by using points to represent people and line segments connecting the points to represent handshakes.



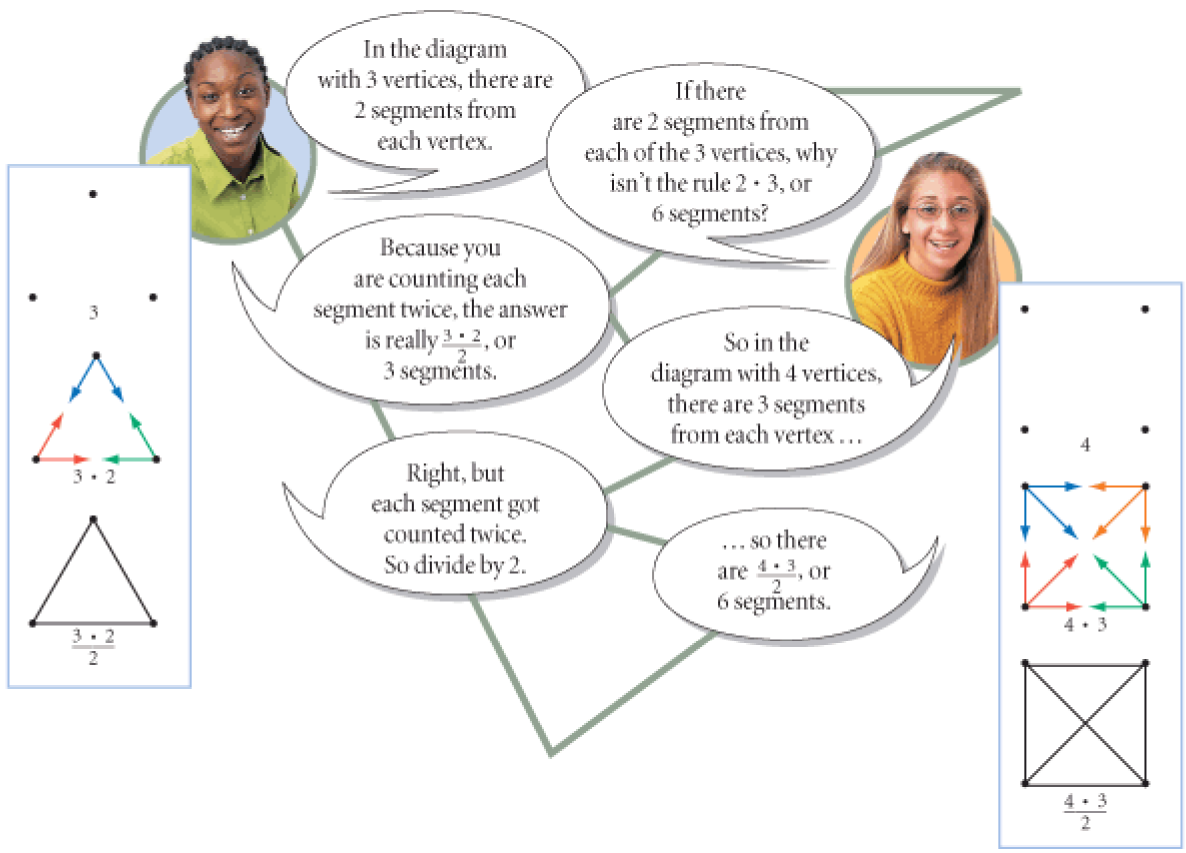
Record your results up to 6 people in the this table.

Number of points (people)	1	2	3	4	5	6	…	*n*	…	30
Number of segments (handshakes)							…		…	

Notice that the pattern does not have a constant difference. That is, the rule is not a linear function. So we need to look for a different kind of rule.



D.) Refer to the table you made for part C. The pattern of differences is increasing by one: 1, 2, 3, 4, 5, 6, 7. Read the dialogue between Erin and Stephanie as they attempt to use logical reasoning to find the rule.



Let's continue with Stephanie and Erin's line of reasoning.

E.) In the diagram with 5 vertices, how many segments are there from each vertex? ______

So the total number of segments written in factored form is $\frac{5\cdot\_\_\_}{2}$.

F.) Complete the table below by expressing the total number of segments in factored form.

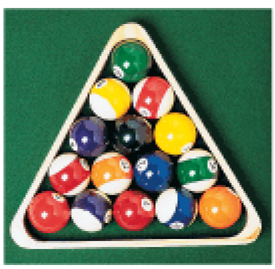
Number of points (people)	1	2	3	4	5	6	…	n
Number of segments (handshakes)	$\frac{(1)(0)}{2}$	$\frac{(2)(1)}{2}$	$\frac{(3)(2)}{2}$	$\frac{(4)(3)}{2}$	$\frac{(5)(\_\_\_)}{2}$	$\frac{(6)(\_\_\_)}{2}$	…	$\frac{(\_\_\_)(\_\_\_)}{2}$

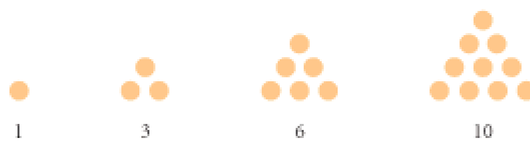
G.) The larger of the two factors in the numerator represents the number of points. What does the smaller of the two numbers in the numerator represent?

Why do we divide by 2?

H.) Write a function rule. How many handshakes were there at the party (of 30 people)?

The numbers in the pattern in the previous investigation are called the **triangular numbers** because you can arrange them into a triangular pattern of dots.

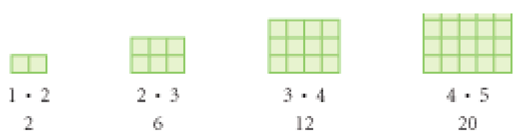




Fifteen pool balls can be arranged in a triangle, so 15 is a triangular number.

The triangular numbers appear in many geometric situations, as you will see in the exercises in the assignment. They are related to this sequence of **rectangular numbers**: 2, 6, 12, 20, 30, 42,…

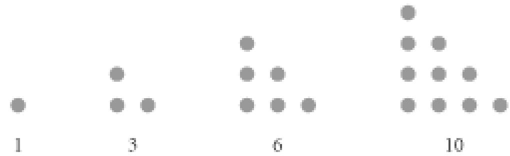
Rectangular numbers can be visualized as rectangular arrangements of objects, in which the length and width are factors of the numbers.



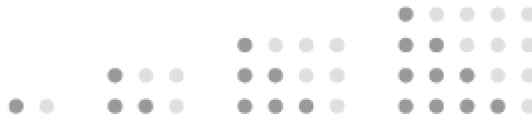
In this sequence, the width is equal to the ______ number and the length is one more than the term number. The rectangle representing the 3rd term, for instance, has width 3 and length 3 + 1, or 4, so the total number of squares is equal to 3 · 4, or _____. You can apply this pattern to find any term in the sequence. The 25th rectangle, for example, would have width _____, length ____, and a total number of squares equal to ____ · _____, or _______.

In general, the *n*th rectangle in this sequence has a width equal to the term number, n, and a length equal to one more than the term number, or $n + 1$. So the *n*th rectangular number is $n(n + 1)$.

Here is a visual approach to arrive at the rule for the party handshake problem. If we arrange the triangular numbers in stacks,



you can see that each is half of a rectangular number.



So the triangular array has $\frac{n(n+1)}{2}$ dots.

⇒**ASSIGNMENT**: ____