Lesson 4.1: Triangle Sum Conjecture

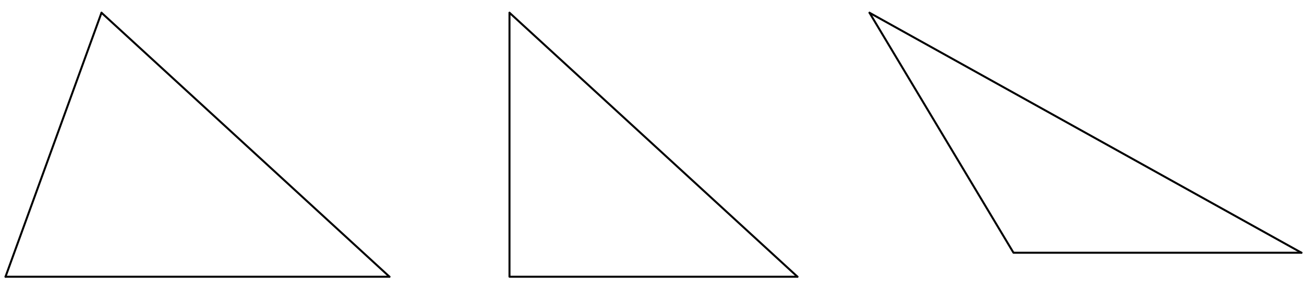
In this lesson you will:
- state a conjecture about the sum of the measures of the angles in a triangle
- complete a paragraph proof of the Triangle Sum Conjecture

Triangles have certain properties that make them useful in all kinds of structures, from bridges to high-rise buildings. One such property of triangles is their rigidity. Another application of triangles is a procedure used in surveying called **triangulation**. This procedure allows surveyors to ___________ points or positions on a map by measuring angles and distances and creating a network of triangles. Triangulation is based on an important property of plane geometry that you will discover in this lesson.

Investigation 4.1: "The Triangle Sum"

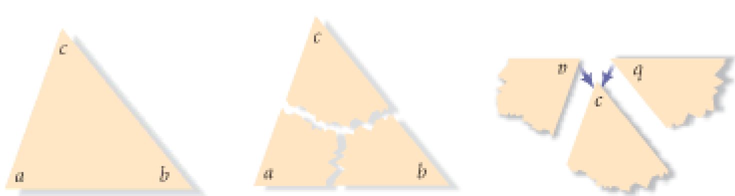
There are an endless variety of triangles that you can draw, with different shapes and angles measures. Do their angle measures have anything in common?

A.) Measure the three angles of each triangle as accurately as possible with your protractor.



B.) Find the sum of the three angles in each triangle. Compare results with others in your group. Does everyone get about the same result? ______ What is the total measure of all three angles in each triangle? _______

C.) There is another way to check the sum. Suppose you have the triangle below. If you were to tear off the three angles and then arrange them so that their vertices meet at a point, how would this arrangement show the sum of the angles measured?



D.) State your observations as a conjecture. (Add to your conjecture list, too.)

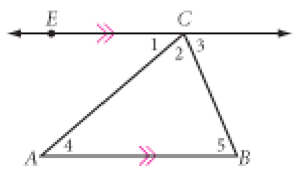
Triangle Sum Conjecture (C-17)

The sum of the measures of the angles in every triangle is _________.

The investigation may have convinced you that the Triangle Sum Conjecture is true, but can you explain *why* it is true for every triangle?

Write a **<u>paragraph proof</u>** (a deductive argument that uses written sentences to support claims with reasons.) In your proof, you can use conjectures, definitions, and properties to support your argument.

One reasoning strategy that you might want to use is to add an **<u>auxiliary line</u>** (an extra line or segment that helps with a proof). The figure below includes $\overleftrightarrow{EC}$, an auxiliary line parallel to side $\overline{AB}$. Use this diagram to discuss the questions below.



- What are you trying to prove?
- What is the relationship among $\angle 1$, $\angle 2$, and $\angle 3$?
- Why was the auxiliary line drawn to be parallel to one of the sides?
- What other congruencies can you determine from the diagram?

Use your responses to these questions to mark your diagram with other relationships that might help with your proof.

<u>Paragraph Proof: The Triangle Sum Conjecture</u>

Consider $\angle 1$ and $\angle 2$ together as a single angle that forms a ________________ with $\angle 3$. By the __________________ Conjecture, their measures must add up to ______.

$$m\angle 1 + m\angle 2 + m\angle 3 = \_\_\_\_\_\_$$

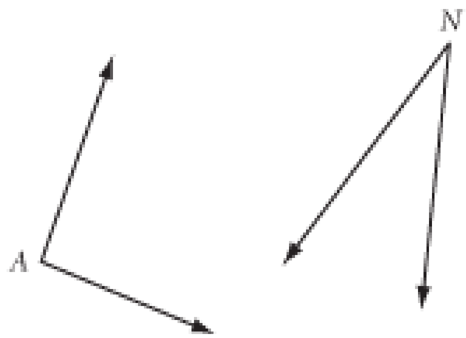
$\overline{AC}$ and $\overline{BC}$ form ________________ between parallel lines $\overleftrightarrow{EC}$ and $\overleftrightarrow{AB}$. By the __________________________________ Conjecture, $\angle 1$ and $\angle 4$ are _____________ and $\angle 3$ and $\angle 5$ are _____________, so their measures are ____________.

$$m\angle 1 \_\_ m\angle 4 \quad \text{and} \quad m\angle 3 \_\_ m\angle 5$$

Substitute $m\angle 4$ for _______ and $m\angle 5$ for ________ in the first equation above.

Therefore, the measures of the angles in a triangle add up to 180°.

•Example 1: Given $\angle A$ and $\angle N$, construct $\angle D$, the third angle of ΔAND.

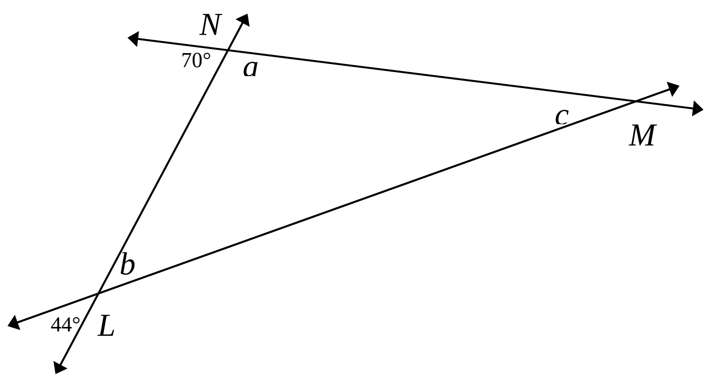


•Example 2: Find the labeled angle measures using what you have learned about angles. (Don't use a protractor; the angles are not exactly to scale.)

a = ______

b = ______

c = ______



⇒**ASSIGNMENT**: ____