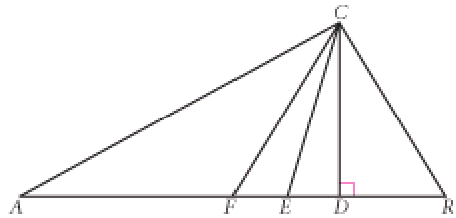
Lesson 4.8: Proving Special Triangle Conjectures

In this lesson you will:

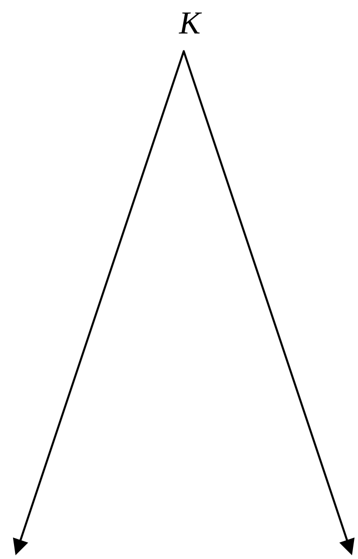
- make a conjecture about the bisector of the vertex angle in an isosceles triangle
- make and prove a conjecture about equilateral triangles
- learn about biconditional conjectures

In ΔARC, $\overline{CD}$ is the altitude to the base $\overline{AR}$, $\overline{CE}$ is the angle bisector of $\angle ACR$, and $\overline{CF}$ is the median to side $\overline{AR}$. This example illustrates that the angle bisector, the altitude, and the median can all be different segments. Is this always true? Can they all be the same segment? You will explore these questions in the investigation.



<u>Investigation 4.8: "The Symmetry Line in an Isosceles Triangle"</u>

A.) On the rays given below construct an isosceles triangle. Label the other two vertices as *R* and *A*.



B.) Construct angle bisector $\overline{KD}$ with point *D* on $\overline{AR}$. Do ΔADK and ΔRDK look congruent? ________ If they are congruent, then $\overline{KD}$ is a line of symmetry.

C.) With your compass, compare $\overline{AD}$ and $\overline{RD}$. Is *D* the midpoint of $\overline{AR}$? ________ If *D* is the midpoint, then what type of special segment is $\overline{KD}$? __________________

D.) Compare $\angle ADK$ and $\angle RDK$. Do they have equal measures? ________ Are they supplementary? ________ What conclusion can you make? _________________________

E.) Combine your observations from parts C and D to complete the conjecture on the back.

Vertex Angle Bisector Conjecture (C-27)

In an isosceles triangle, the bisector of the vertex angle is also the ________________ and the ________________ to the base.

The properties that you just discovered for isosceles triangles also apply to equilateral triangles. Equilateral triangles are also isosceles, although isosceles triangles are not necessarily equilateral.

You have probably noticed the following property of equilateral triangles: When you construct an equilateral triangle, each angle measures 60°. If each angle measures 60°, then all three angles are congruent. So, if a triangle is equilateral, then it is equiangular. This is called the ________________ Triangle Conjecture.

We can write the proof for this conjecture. We need to show that if $AB = AC = BC$, then ΔABC is equiangular.

If $AB = AC$, then _________ = _________ by the ______________________________

If $AB = BC$, then _________ = _________ by the ______________________________

If _________ = _________ and _________ = _________,

then _________ = _________ = _________

Therefore, ____

The converse of the Equilateral Triangle Conjecture is called the Equiangular Triangle Conjecture, and it states: If a triangle is equiangular, then it is equilateral. Is this true? ______

If both are true, we can combine them. Complete the conjecture below, and add it to your conjecture list.

Equilateral/Equiangular Conjecture (C-28)

Every equilateral triangle is ____________________ and, conversely, every equiangular triangle is _____________________.

⇒**ASSIGNMENT**: ____