Lesson 5.4: Properties of Midsegments

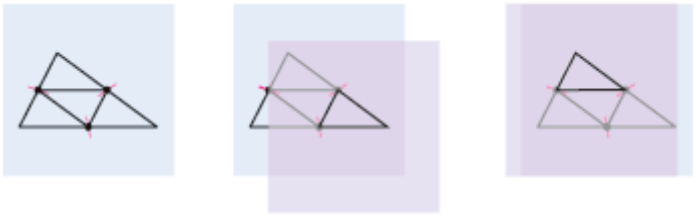
In this lesson you will:

- discover properties of the midsegment of a triangle
- discover properties of the midsegment of a trapezoid

As you learned in Chapter 3, the segment connecting the midpoints of two sides of a triangle is a midsegment of a triangle. The segment connecting the midpoints of the two _____ sides of a trapezoid is also called the midsegment of the trapezoid. In this lesson you will discover special properties of midsegments.

Investigation 5.4.1: "Triangle Midsegment Properties"

In this investigation you will discover two properties of triangle midsegments. Each person in your group can investigate a different triangle.



Step 1 Step 2 Step 3

A.) Draw a triangle on a piece of patty paper. Pinch the patty paper to locate midpoint of the sides. Draw the midsegments. You should now have four small triangles. (See Step 1 in the diagram above.)

B.) Place a second piece of patty paper over the first and copy one of the four small triangles. (See Step 2 of the diagram above.)

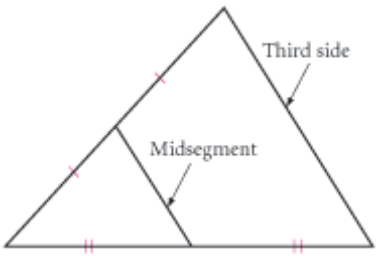
C.) Compare all four triangles by sliding the copy of one small triangle over the other three triangles. What do you notice about all 4 of the small triangles?

D.) Complete the conjecture below, and add it to your conjecture list.

Three Midsegments Conjecture (C-41)

The three midsegments of a triangle divide it into _____.

E.) Redraw your original triangle below with just one midsegment, as in the diagram at right. Mark all the congruent angles. Using the Corresponding Angles Conjecture or its converse, what conclusions can you make about a midsegment and the large triangle's third side?



F.) Compare the length of the midsegment to the large triangle's third side. How do the lengths compare?

G.) Based on your observations, complete the conjecture below, and add it to your conjecture list.

Triangle Midsegment Conjecture (C-42)

A midsegment of a triangle is _____ to the third side and _____ the length of the third side.

Investigation 5.4.2: "Trapezoid Midsegment Properties"

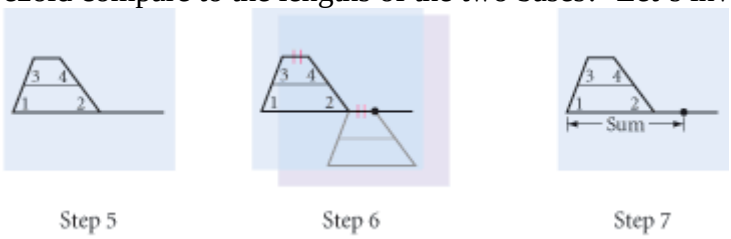
Each person in your group can investigate a different trapezoid.

A.) Draw a small trapezoid on the left side of a piece of patty paper. (Be sure you draw the bases perfectly parallel by using the both sides of your ruler as guides.) Pinch the paper to locate the midpoints of the nonparallel sides. Draw the midsegment. (See Step 1 of the diagram above.)

B.) Label the angles on your trapezoid as shown in Step 2's diagram above. Place a second piece of patty paper over the first and copy the trapezoid and its midsegment.

C.) Compare the trapezoid's base angles with the corresponding angles at the midsegment by sliding the copy up over the original. (See Step 3 above.) Are the corresponding angles congruent? _____ What can you conclude about the midsegment and the bases?

The midsegment of a triangle is half the length of the third side. How does the length of the midsegment of a trapezoid compare to the lengths of the two bases? Let's investigate.



D.) On your original trapezoid, extend the longer base to the right by at least the length of the shorter base. (See Step 5 of the diagram above.)

E.) Slide the second patty paper under the first. Show the sum of the lengths of the two bases by marking a point on the extension of the longer base. (See Step 6.)

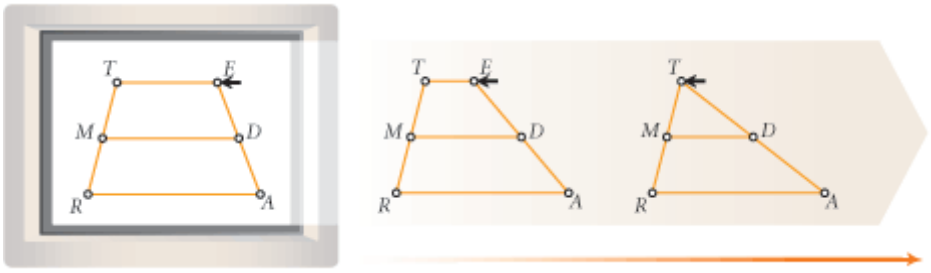
F.) How many times does the midsegment fit onto the segment representing the sum of the lengths of the two bases? _____ What do you notice about the length of the midsegment and the sum of the lengths of the two bases?

G.) Combine your conclusions from parts C and F and complete the conjecture below, and then add it to your conjecture list.

Trapezoid Midsegment Conjecture (C-43)

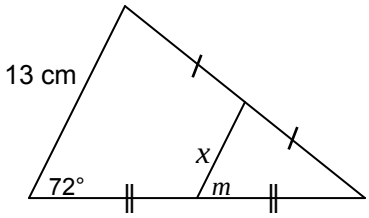
The midsegment of a trapezoid is _____ to the bases and is _____ in length to the average of the lengths of the bases.

What happens if one base of the trapezoid shrinks to a point? Then the trapezoid collapses into a triangle, the midsegment of the trapezoid becomes a midsegment of the triangle, and the Trapezoid Midsegment Conjecture becomes the Triangle Midsegment Conjecture. Do both of your midsegment conjectures work for the last figure? _____

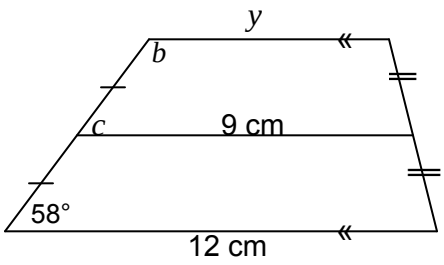


•Example 1: Find the lettered measures.

a.)



b.)



⇒**ASSIGNMENT**: _____