Lesson 6.2: Chord Properties

In this lesson you will:

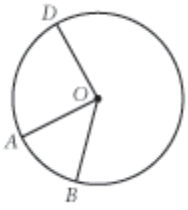
- define central angle and inscribed angle
- investigate properties of chords of a circle

In the last lesson you discovered some properties of a tangent, a line that intersects the circle only _____. In this lesson you will investigate properties of a chord, a line segment whose _____ lie on the circle.

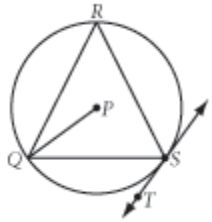
Investigation 6.2.1: "Defining Angles in a Circle"

Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions for your class and add them to your vocabulary list.

Central Angle

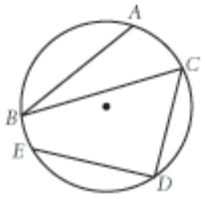


$\angle AOB$, $\angle DOA$, and $\angle DOB$ are central angles of circle O.

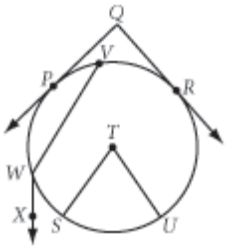


$\angle PQR$, $\angle PQS$, $\angle RST$, $\angle QST$, and $\angle QSR$ are NOT central angles of circle P.

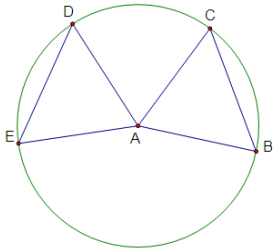
Inscribed Angle



$\angle ABC$, $\angle BCD$, and $\angle CDE$ are inscribed angles.



$\angle PQR$, $\angle STU$, and $\angle VWX$ are NOT inscribed angles.



Investigation 6.2.2: "Chords and Their Central Angles"

Next you will discover some properties of chords and central angles. You will also see a relationship between chords and arcs.

***Open "Lesson 6.2 (Chord Properties) Geo Sketchpad.gsp" and go to the "Inv 2" tab/page.**

A.) What is true about chords $\overline{BC}$ and $\overline{DE}$?

B.) Drag different parts of your figure to confirm that the chords stay congruent.

C.) Using the segment tool, construct segments $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, and $\overline{AE}$. 

D.) Measure central angles $\angle CAB$ and $\angle DAE$. (Remember to measure an angle, make sure the vertex is the second point that you select.) How do the central angles compare?

E.) Drag different parts of your figure. Is your statement in part D still true? _____

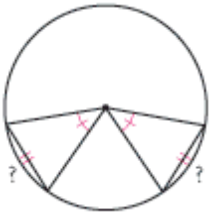
F.) Share your results with others in your group. Then complete the conjecture below.

Chord Central Angles Conjecture (C-55)

If two chords in a circle are congruent, then they determine two central angles that are _____.

G.) Recall that the measure of an arc is defined as the measure of its central angle. Select point B, point C, and the circle, and choose **Measure | Arc Angle** to confirm this.

H.) Select point D, point E, and the circle, and choose **Measure | Arc Angle**.

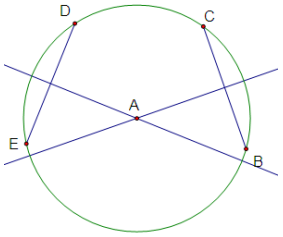


I.) How do the intercepted arcs $\overset{\frown}{BC}$ and $\overset{\frown}{DE}$ compare?

J.) Complete the conjecture below based on your observation from part I.

Chord Arcs Conjecture (C-56)

If two chords in a circle are congruent, then their _____ are congruent.



Investigation 6.2.3: "Chords and the Center of the Circle"

In this investigation you will discover relationships about a chord and the center of its circle.

*__Go to the tab/page for "Inv 3."__

A.) Select chord $\overline{BC}$ and point A, and choose **Construct | Perpendicular Line**.

B.) Select chord $\overline{DE}$ and point A, and choose **Construct | Perpendicular Line**.

C.) Drag point C and observe the relationship between the chord and the line perpendicular to it from the center of the circle.

D.) How does the perpendicular from the center of a circle to a chord divide the chord?

E.) Based on this observation, complete the conjecture below.

Perpendicular to a Chord Conjecture (C-57)

The perpendicular from the center of a circle to a chord is the _____ of the chord.

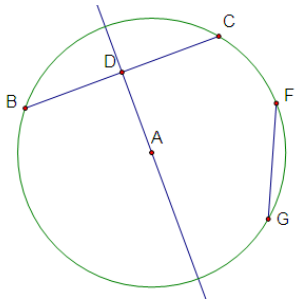
F.) Measure the distance from point A to $\overline{BC}$ and the distance from point A to $\overline{DE}$. (Select a point and a segment, and choose **Measure | Distance**.) How do the distances compare?

G.) Drag parts of your sketch and observe these distances. Are the results the same if you change the size of the circle and the length of the chords? _____

H.) Based on your observations, complete the next conjecture.

Chord Distance to Center Conjecture (C-58)

Two congruent chords in a circle are _____ from the center of the circle.



Investigation 6.2.4: "Perpendicular Bisector of a Chord"

Next, you'll discover a property of perpendicular bisectors of chords.

*__Go to the tab/page for "Inv 4."__

A.) Construct the midpoint of chord $\overline{BC}$. (Select segment $\overline{BC}$, and choose **Construct | Midpoint**.)

B.) Label the midpoint D.

C.) Construct a line through point D, perpendicular to $\overline{BC}$. (Select point D and segment $\overline{BC}$, and choose **Construct | Perpendicular Line**.)

D.) Construct the midpoint of chord $\overline{FG}$. (Select segment $\overline{FG}$, and choose **Construct | Midpoint**.)

E.) Label the midpoint H.

F.) Construct a line through point H, perpendicular to $\overline{FG}$. (Select point H and segment $\overline{FG}$, and choose **Construct | Perpendicular Line**.)

G.) What do you notice about the point of intersection of the two perpendicular bisectors? (Drag parts of your sketch to confirm that this is always true.)

H.) Compare your result with the results of your group members. Based on your observations, complete the conjecture below.

Perpendicular Bisector of a Chord Conjecture (C-59)

The perpendicular bisector of a chord passes through the _____ of the circle.

⇒**ASSIGNMENT**: _____