Lesson 6.3: Arcs and Angles

In this lesson you will:

- make conjectures about inscribed angles in a circle
- investigate relationships among the angles in a cyclic quadrilateral
- compare the arcs formed when two parallel lines intersect a circle

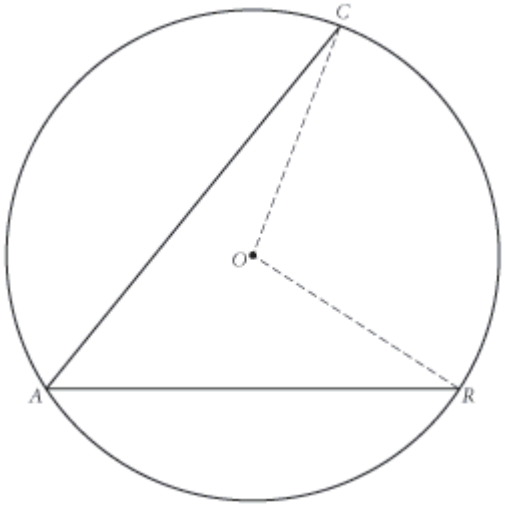
Many arches that you see in structures are semicircular, but Chinese builders long ago discovered that arches don't have to be this shape. The Zhaozhou bridge, shown to the right, was completed in 605 C.E. It is the world's first stone arched bridge in the shape of a minor arc, predating other minor-arc arches by about 800 years.

In this lesson you'll discover properties of arcs and the angles associated with them.

Investigation 6.3.1: "Inscribed Angle Properties"

In this investigation you will compare an inscribed angle and a central angle, both inscribed in the same arc. Refer to the diagram of circle *O*, with center angle *COR* and inscribed angle *CAR*.

A.) Measure $\angle COR$ with your protractor to find $m\overset{\frown}{CR}$.
_____ Measure $\angle CAR$. _____ How does the $m\angle CAR$ compare with the $m\overset{\frown}{CR}$?



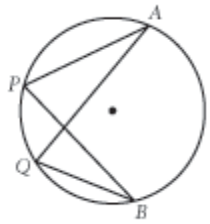
B.) Construct a circle of your own with an inscribed angle. Draw and measure the central angle that intercepts the same arc. What is the measure of the central angle? _____ What is the measure of the inscribed angle? _____ How do the two measures compare?

C.) Based on your observations, complete the conjecture below.

Inscribed Angle Conjecture (C-60)

The measure of an angle inscribed in the circle is _____ the measure of the intercepted arc.

Investigation 6.3.2: "Inscribed Angles Intercepting the Same Arc"



Next, let's consider two inscribed angles that intercept the same arc. In the figure at right, $\angle AQB$ and $\angle APB$ both intercept arc *AB*. Angles *AQB* and *APB* are both inscribed in $\overset{\frown}{APB}$.

A.) Construct a large circle. Select two points on the circle. Label them *A* and *B*. Select a point P on the major arc and construct inscribed angle *APB*. With your protractor, measure $\angle APB$. _____

B.) Select another point *Q* on $\overset{\frown}{APB}$ and construct inscribed angle *AQB*. Measure $\angle AQB$. _____

C.) How does $m\angle APB$ compare with $m\angle AQB$.

D.) Now pick points *X* and *Y* on minor arc *AB*. Measure $\angle AXB$ and $\angle AYB$. How do these measures compare?

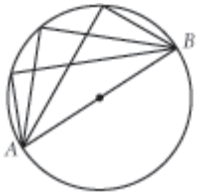
E.) Based on your observations, complete the conjecture below.

Inscribed Angles Intercepting Arcs Conjecture (C-61)

Inscribed angles that intercept the same arc are _____.

Investigation 6.3.3: "Angles Inscribed in a Semicircle"

Next you will investigate a property of angles inscribed in semicircles. This will lead you to a third important conjecture about inscribed angles.



A.) Construct a large circle. Construct diameter $\overline{AB}$. Inscribe three angles in the same semicircle. Make sure the sides of each angle pass through *A* and *B*.

B.) Measure each angle with your protractor. What do you notice?

C.) Based on your observations, complete the conjecture below.

Angles Inscribed in a Semicircle Conjecture (C-62)

Angles inscribed in a semicircle are _____ angles.

Investigation 6.3.4: "Cyclic Quadrilaterals"

*Add "cyclic quadrilateral" to your vocabulary list.

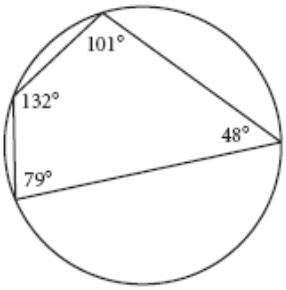
A quadrilateral inscribed in a circle is called a cyclic quadrilateral. Each of its angles is inscribed in the circle, and each of its sides is a chord or the circle.

A.) Construct a large circle. Construct a cyclic quadrilateral by connecting four points anywhere on the circle.

B.) Measure and label each of the four inscribed angles.

C.) Find the sums of pairs of consecutive angles and sums of pairs of opposite angles. What do you discover?

D.) At right is another example of a cyclic quadrilateral. Find the sums of pairs of consecutive angles and sums of pairs of opposite angles. What do you discover?

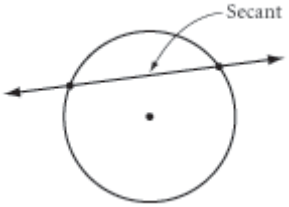


E.) Based on your observations, complete the conjecture below.

Cyclic Quadrilateral Conjecture (C-63)

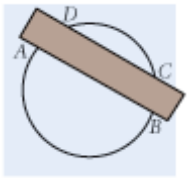
The _____ angles of a cyclic quadrilateral are _____.

Investigation 6.3.5: "Arcs by Parallel Lines"



*Add "secant" to your vocabulary list.

A line that intersects a circle in two points is called a secant. A secant contains a chord of the circle, and passes through the interior of a circle, while a tangent line does not. Note that a secant is a line while a chord is a segment.



A.) On a piece of patty paper, construct a large circle. Lay your straightedge across the circle so that its parallel edges pass through the circle. Draw secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ along both edges of the straightedge.

B.) Fold your patty paper to compare $\overline{AD}$ and $\overline{BC}$. What can you say about $\overline{AD}$ and $\overline{BC}$?

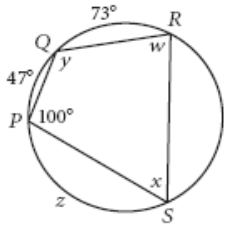
C.) Repeat parts A and B, using lined paper to construct different parallel secants.

D.) Based on your observations, complete the conjecture below.

Parallel Lines Intercepted Arcs Conjecture (C-64)

Parallel lines intercept _____ arcs on a circle.

•Example 1: Find each lettered measure.



⇒**ASSIGNMENT**: _____