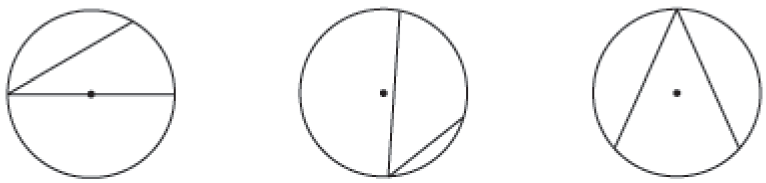
Lesson 6.4: Proving Circle Conjectures

In this lesson you will:
- prove the Inscribed Angle Conjecture
- prove other conjectures related to circles

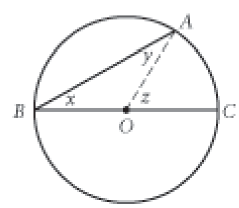
In Lesson 6.3, you discovered the Inscribed Angle Conjecture: The measure of an inscribed angle in a circle equals _________ the measure of its intercepted arc. In this lesson, you will prove this conjecture. But how? Let's use our reasoning strategies to make a plan. By thinking backward, we see that a central angle gives us something to compare an inscribed angle with. If one side of the inscribed angle is a diameter, then we can form a central angle by adding an auxiliary line. But what if the circle's center is not on the inscribed angle? There are three possible cases.



Case 1 — The circle's center is on the angle.
Case 2 — The center is outside the angle.
Case 3 — The center is inside the angle.

Let's break the problem into parts and consider one case at a time. We'll start with the easier case first.

Case 1: The circle's center is on the inscribed angle.
This proof uses the variables *x*, *y*, and *z* to represent the measures of the angles as shown in the diagram at right.



Given: Circle *O* with inscribed angle *ABC* on diameter $\overline{BC}$

Show: $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

Flowchart Proof of Case 1

$\overline{AO} \cong$ ____
All radii of a circle are congruent.

ΔAOB is ____________
____

$x = y$
____

____
Triangle Exterior Angle Conj.

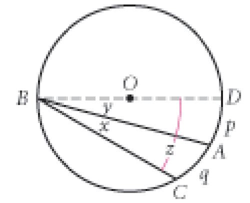
$x + x = z$ Substitute for y
$2x = z$ Combine like terms
$x = ½\, z$ Divide both sides by 2
Properties of Equality

$x = m\angle ABC$
given

$z = m\overset{\frown}{AC}$
Def. of arc measure

[ ]
____

Case 2: The circle's center is outside the inscribed angle.
This proof uses the variables *x*, *y*, and *z* to represent the measures of the angles, and *p* and *q* to represent the measures of the arcs, as shown in the diagram at right.



Given: Circle *O* with inscribed angle *ABC* on one side of diameter $\overline{BC}$

Show: $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

Flowchart Proof of Case 2

$y = ½\ m\overset{\frown}{DA}$
Proof of Case 1

$z = ½\ m\overset{\frown}{DC}$
Proof of Case 1

$p + \_\_\_\_ = m\overset{\frown}{DC}$
Arc Addition

$x + y = \_\_\_\_\_$
Angle Addition

$y = \frac{1}{2}\_\_\_\_\_$
Substitution

$z = \frac{1}{2}\_\_\_\_\_\_\_\_\_\_$
Substitution

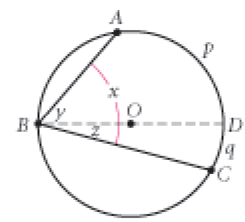
$x + ½p = ½(p + q)$ Substitute for y and z
$x + ½p = \_\_\_\_\_\_\_\_\_\_\_\_$ Distribute the ½
$x = \_\_\_\_\_\_$ Subtract ½ p from both sides
Properties of Equality

$x = m\angle ABC$
Given

$q = m\overset{\frown}{AC}$
Given

Substitution

Case 3: The circle's center is inside the inscribed angle.
This proof uses the variables *x*, *y*, and *z* to represent the measures of the angles, and *p* and *q* to represent the measures of the arcs, as shown in the diagram at right.



Given: Circle *O* with inscribed $\angle ABC$ whose sides lie on either side of diameter $\overline{BC}$

Show: $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

Flowchart Proof of Case 3

$y = ½\ m\overset{\frown}{DA}$
Proof of Case 1

$z = ½\ m\overset{\frown}{DC}$
Proof of Case 1

$p + \_\_\_\_ = m\overset{\frown}{AC}$
Arc Addition

$x = \_\_\_\_\_\_\_\_\_\_$
Angle Addition

$y = \frac{1}{2}\_\_\_\_\_$
Substitution

$z = \frac{1}{2}\_\_\_\_\_\_$
Substitution

$x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ Substitute for y and z
$x = \_\_\_\_\_\_\_\_\_\_\_$ Factor ½ from each term
Properties of Equality

$x = \_\_\_\_\_\_\_\_\_\_\_\_$
Given

$x = m\angle ABC$
Given

Substitution

⇒**ASSIGNMENT**: ____