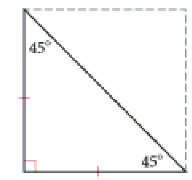
Lesson 9.3: Two Special Right Triangles

In this lesson you will:
- discover a shortcut for finding an unknown side length in an isosceles right triangle (also called a 45°-45°-90° triangle)
- discover a shortcut for finding an unknown side length in a 30°-60°-90° triangle

In this lesson you will use the Pythagorean Theorem to discover some relationships between the sides of two special right triangles.

One of these special triangles is an isosceles right triangle, also called a 45°-45°-90° triangle. Each isosceles triangle is half a ____________, so these triangles show up often in mathematics and engineering.

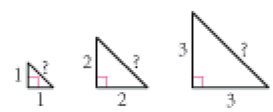


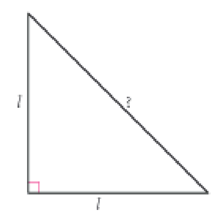
An isosceles right triangle

Investigation 9.3.1: "Two Special Right Triangles"

In this investigation you will simplify radicals to discover a relationship between the length of the legs and the length of the hypotenuse in a 45°-45°-90° triangle. To simplify a square root means to write it as a multiple of a smaller radical <u>without</u> using decimal approximations.

A.) Find the length of the hypotenuse of each isosceles right triangle at right. Simplify each square root.





B.) Use your results to complete the following table. Draw additional triangles as needed.

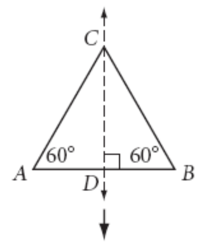
Length of each leg	1	2	3	4	5	6	7	…	10	…	l
Length of hypotenuse								…		…	

C.) Use your results to complete the following conjecture.

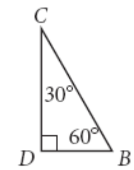
Isosceles Right Triangle Conjecture (C-83)

In an isosceles right triangle, if the legs have length l, then the hypotenuse has length ________.

Another special triangle is a 30°-60°-90° triangle, also called a 30°-60° right triangle, that is formed by bisecting any angle of an equilateral triangle. The 30°-60°-90° triangle also shows up often in mathematics and engineering because it is half of an ___________________ triangle. Let's prove it.



Developing Proof: Create a flowchart proof that the angle bisector through angle C in equilateral triangle ABC at right forms two congruent triangles, ΔACD and ΔBCD.



ΔABC is equilateral →

$\overleftrightarrow{CD}$ bisects $\angle C$ →

Additional questions about the 30°-60°-90° triangle:

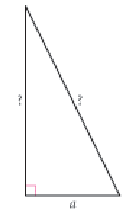
1. Why must the angles in ΔBCD be 30°, 60°, and 90°?

2. How does BD compare to AB? How does BD compare to BC?

3. In any 30°-60°-90° triangle, how dos the length of the hypotenuse compare to the length of the shorter leg?

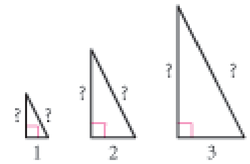
Let's use this relationship between the shorter leg and the hypotenuse of a 30°-60°-90° triangle and the Pythagorean Theorem to discover another relationship.

Investigation 9.3.2: “30°-60°-90° Triangles”



In this investigation you will simplify radicals to discover a relationship between the lengths of the shorter and longer legs in a 30°-60°-90° triangle.

A.) Use the relationship from the developing proof activity above to find the length of the hypotenuse of each 30°-60°-90° triangle at right. Then use the Pythagorean Theorem to calculate the length of the third side. Simplify each square root. Use your results to fill in the table below.



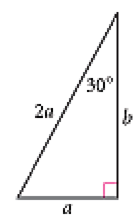
Length of shorter leg	1	2	3	4	5	6	7	…	10	…	*a*
Length of hypotenuse								…		…	
Length of longer leg								…		…	

B.) Using your results complete the conjecture below.

30°-60°-90° Triangle Conjecture (C-84)

In a 30°-60°-90° triangle, if the shorter leg has length *a*, then the longer leg has length ______ and the hypotenuse has length ______.

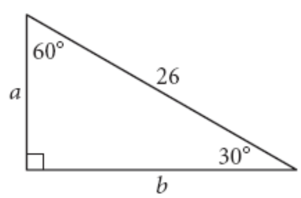
You can use algebraic symbols to verify the 30°-60°-90° Triangle Conjecture.



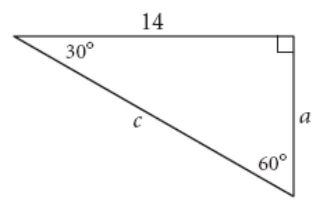
$$a^2 + b^2 = c^2$$

•Example 1: Find the lettered side lengths. All lengths are in centimeters.

a.



b.



⇒**ASSIGNMENT**: ____