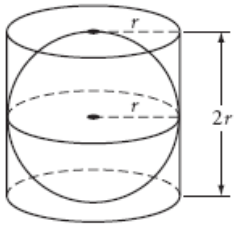
Lessons 10.6 & 10.7: Volume and Surface Area of a Sphere

In this lesson you will:

- discover the volume formula for a sphere
- apply volume and surface area formulas to problems involving spheres or hemispheres

You can find the volume of a sphere by comparing it to the volume of a right cylinder.

Investigation 10.6: "The Formula for the Volume of Sphere"



This investigation demonstrates the relationship between the volume of a sphere with radius r and the volume of a right cylinder with base radius r and height $2r$—that is, the smallest cylinder that encloses a given sphere.

A.) The height of the smallest cylinder that encloses a sphere is $2r$. Why is this true?

B.) If the radius of the cylinder is r and its height is $2r$, then what is the volume of the cylinder in terms of r?

$V =$ _____

C.) Make a sphere out of playdoh. Wrap a piece of transparency around the sphere to make a "cylinder" that encloses the sphere. Be sure that the height of the "cylinder" is equal to the diameter of the sphere. What is the height of the cylinder? _____ cm What is the radius of the cylinder (and the sphere)? _____ cm

D.) If you change the shape of the playdoh sphere by flattening it out, will the volume of the playdoh change? _____

E.) Squash the playdoh down into the "cylinder." Now the sphere is a cylinder. Has the radius changed? _____ What is the new height of the playdoh cylinder? _____ cm

F.) The playdoh "cylinder" (the flattened sphere) is approximately what fraction of the transparency cylinder?

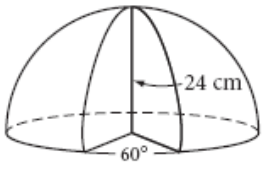
G.) The volume of a sphere is the fraction (from part F) of the original cylinder's volume (that you found in part B). Based on this information, complete the conjecture below.

Sphere Volume Conjecture (C-88)

The volume of a sphere with radius r is given by the formula $V =$ _____.

- Example 1: Find the volume of a sphere with a radius of 4 cm.

- Example 2: Find the volume of this solid.



- Example 3: A marble is submerged in water in a rectangular prism with 2 cm-by-2 cm base. The water in the prism rises 0.9 cm when the marble is submerged. What is the diameter of the marble?

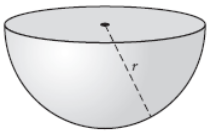
In chapter 8, we found the surface area of prisms, pyramids, cylinders and cones, but not spheres. We will now add the formula for the surface area of a sphere to our conjecture list.

Sphere Surface Area Conjecture (C-89)

The surface area, *S*, of a sphere with radius *r* is given by the formula $S = 4\pi r^2$.

•Example 4: Find the surface area of a sphere with a radius of 4 cm.

•Example 5: The base of this hemisphere has circumference 32π cm. Find the surface area of the hemisphere (including the base).



•Example 6: Find the surface area of a sphere whose volume is $12{,}348\pi$ m^3.

⇒**ASSIGNMENT**: _____