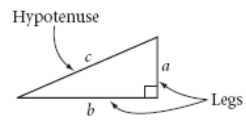
Lessons 9.1 & 9.2: The Theorem of Pythagoras

In this lesson you will:

- review the Pythagorean Theorem, which states the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle.
- determine if the converse of the Pythagorean Theorem is true.
- use the Pythagorean Theorem (and the converse) to solve problems.

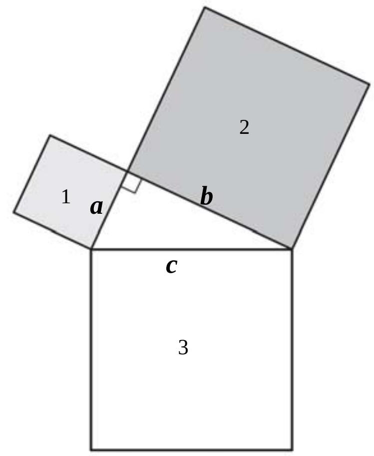
In a right triangle, the side opposite the right angle is called the _________________. The other two sides are called ________. In the figure, a and b are the lengths of the legs of a right triangle, and c is the length of the hypotenuse. There is a special relationship between the lengths of the legs and the length of the hypotenuse. This relationship is known as the _________________ Theorem.



A theorem is a conjecture that has been ____________.

*Add “hypotenuse” and “theorem” to your vocabulary list.

In eighth grade, you explored the relationship between a, b, and c, using diagrams like the one at right.



♦In the diagram at right, what is the area of the square labeled 1? _____

♦What is the area of the square labeled 2? _____

♦What is the area of the square labeled 3? _____

Do you remember the relationship between a, b, and c? What is it? _______________
Is this true for every right triangle? ______

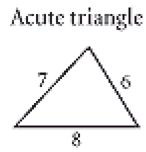
Use that relationship to complete the Pythagorean Theorem.

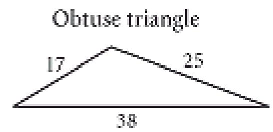
The Pythagorean Theorem (C-81)

In a right triangle, the sum of the squares of the lengths of the legs equals the ___________ of the length of the __________________. (Or in other words, $a^2 + b^2 =$ _____ where a and b are the lengths of the ______ and c is the length of the __________________.)

There are over 200 different proofs of the Pythagorean Theorem! In addition to Pythagoras, other well-known people that have developed original proofs of this theorem include Euclid, Leonardo da Vinci, and U.S. president James Garfield.

The Pythagorean Theorem works for right triangles, but does it work for all triangles? Check the following acute and obtuse triangles below to see if the relationship holds.



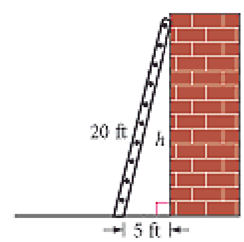


$6^2 + 7^2 \; \square \; 8^2$ $17^2 + \; \square \; 25^2 \quad 38^2$

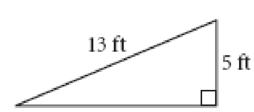
Does the Pythagorean Theorem work for triangles other than right triangles? ______

•Example 1: An Olympic soccer field is a rectangle 100 meters long and 70 meters wide. How long is the diagonal of the field.

•Example 2: How high up on the wall will a 20-foot ladder touch if the foot of the ladder is placed 5 feet from the wall?



•Example 3: What is the area of a right triangle with a leg of length 5 feet and a hypotenuse of length 13 feet?



We know that the Pythagorean Theorem is true but what about the converse of the Pythagorean Theorem? If *x*, *y*, and *z* are the lengths of the three sides of a triangle and they satisfy the Pythagorean equation, $a^2 + b^2 = c^2$, must the triangle be a right triangle?

Intuitively, do you think that the converse is true? _____

Let's look at a quick demonstration to check if a Pythagorean triple (a set of three positive integers that satisfy the Pythagorean Theorem) gives us a right triangle.

Converse of the Pythagorean Theorem (C-82)

If the lengths of three sides of a triangle satisfy the Pythagorean equation ($a^2 + b^2 = c^2$), then the triangle is a ________________________.

•Example 4: Is a triangle with sides measuring 12 cm, 16 cm, and 20 cm a right triangle?

•Example 5: Les wants to build a rectangular pen for his guinea pig. When he finished, he measured the bottom of the pen. He found that one side was 54 inches long, the adjacent side was 30 inches long, and one diagonal was 63 inches long. Is the pen really rectangular?

⇒**ASSIGNMENT**: ____